ASTRO
IIT KANPUR

A DEEP DIVE INTO THE REALMS OF ASTRONOMY, THE WONDERS OF SPACE-TIME AND A MYRIAD OF UNANSWERED QUESTIONS.

RAD

PROJECT

ASTRONOMY CLUB IITK

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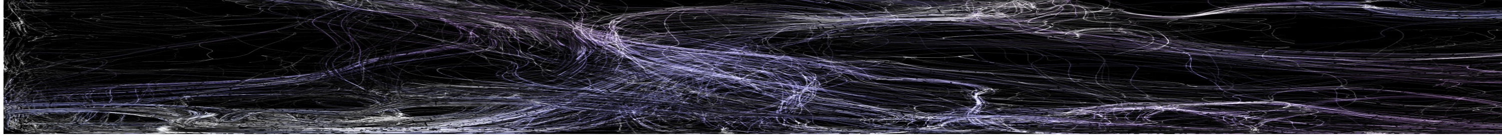

The BIG BANG

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1 Introduction

Cosmology has been there since a time unknown, it is what started other sciences in the first place. The urge to explain our existential dilemma has haunted the human minds since we had a mind- capable of complex thought. Among all of the questions that man had asked after looking up in the heaven, one stands greatest and oldest →

"How did it all Begin"

Our ancestors explained origins from experience, how else could have they done! Thus, came about various epics and tales across various religions and cultures around the world, about the creation and what lies in the fate of their people. Most of these stories arose in part to propitiate and control.

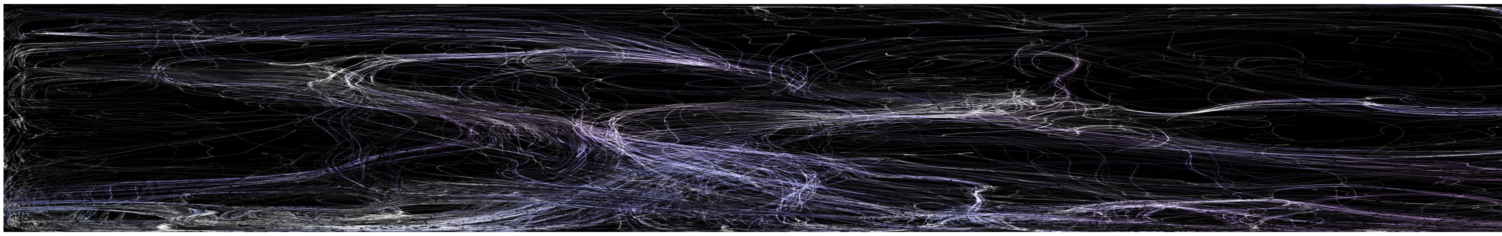
> *"We've tended in our cosmologies to make things familiar. Despite all our best efforts, we've not been very inventive. In the West, Heaven is placid and fluffy, and Hell is like the inside of a volcano. In many stories, both realms are governed by dominance hierarchies headed by gods or devils. Monotheists talked about the king of kings. In every culture we imagined something like our own political system running the Universe. Few found the similarity suspicious."* - Carl Sagan

Then science came along and taught us that we are not the measure of all things, that there are things un-imagined. This is the story that tells us that we are a part of cosmos - that we are the cosmos.

2 Cosmological Principle

The cosmological principle is usually stated formally as 'Viewed on a sufficiently large scale, the properties of the universe are the same for all observers.' This amounts to the strongly philosophical statement that the part of the universe which we can see is a fair sample, and that the same physical laws apply throughout. In essence, this in a sense says that the universe is knowable and is playing fair with scientists. Here "Observers" means any observer at any location in the universe, not simply any human observer at any location on Earth. It can also be put as "the cosmological principle means that the universe looks the same whoever and wherever you are."

It basically contains homogeneity and isotropy of universe on sufficiently large scale. Homogeneity means that the same observational evidence is available to observers at different locations in the universe ("the part of the universe which we can see is a fair sample"). Isotropy means that the same observational evidence is available by looking in any direction in the universe ("the same physical laws apply throughout").



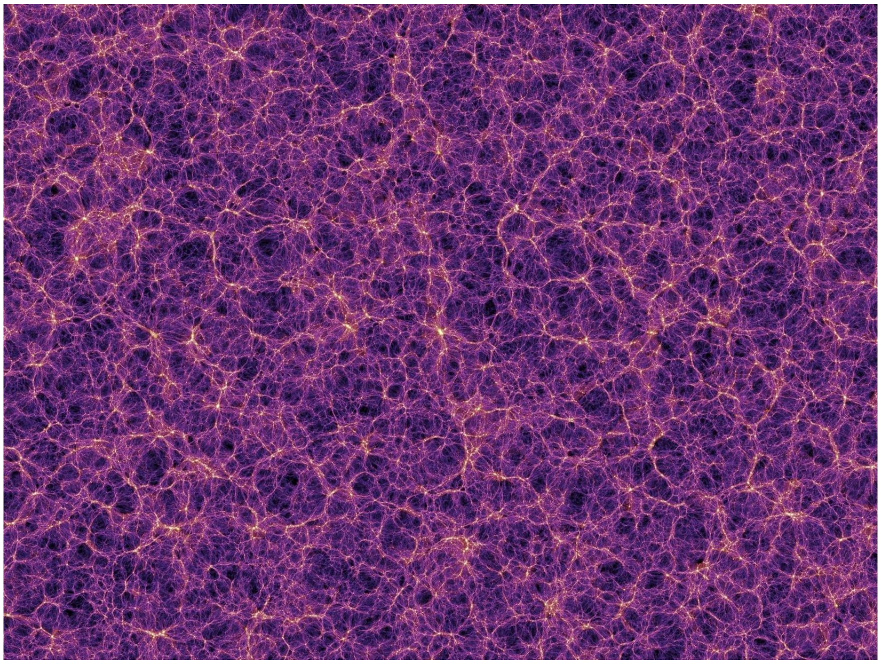


Figure 1: The large scale structure of the universe

3 Perfect cosmological principle

The perfect cosmological principle is an extension of the cosmological principle, and states that the universe is homogeneous and isotropic in space and time. In this view the universe looks the same everywhere (on the large scale), the same as it always has and always will.

4 Steady State Theory

To his great chagrin Einstein found in 1917 that with his three adopted assumptions, his equations of general relativity—as originally written down—had no meaningful solutions. To obtain a solution, Einstein realized that he had to add to his equations an extra term, which came to be called the cosmological constant. If one speaks in Newtonian terms, the cosmological constant could be interpreted as a repulsive force of unknown origin that could exactly balance the attraction of gravitation of all the matter in Einstein's closed universe and keep it from moving. The inclusion of such a term in a more general context, however, meant that the universe in the absence of any mass-energy (i.e., consisting of a vacuum) would not have a space-time structure that was flat (i.e., would not have satisfied the dictates of special relativity exactly). Einstein was prepared to make such a sacrifice only very reluctantly.

As this led to two possibilities which were as follow: -

- On a whole the large structures in universe are stationary with the cosmological constant causing the universe to be stable and as predicted by Einstein. But when Hubble discovered expansion of universe this led to end of this possibility. when Einstein later learned of Hubble's discovery of the expansion of the universe and realized that he could have predicted it had he only had more faith in the original form of his equations, he regretted the introduction of the cosmological constant as the "**biggest blunder**" of his life.
- The only possibility to incorporate perfect cosmological principal and expansion of universe demanded spontaneous creation of matter. The allure of perfect cosmological principal was so much that many famous astronomers and physicians like Bondi, Gold and Hoyle were ready to forgo conservation of mass-energy in order to get the steady state theory.

This theory however continued till two major blows came: -

- The first blow was delivered by British astronomer Martin Ryle's counts of extra-galactic radio sources during the 1950s and '60s. These counts involved the same methods discussed above for the star counts by Dutch astronomer Jacobus Kapteyn and the galaxy counts by Hubble except that radio telescopes were used. Ryle found more radio galaxies at large distances from Earth than can be explained under the assumption of a uniform spatial distribution no matter which cosmological model was assumed, including that of steady state. This seemed to imply that radio galaxies must evolve over time in the sense that there were more powerful sources in the past (and therefore observable at large distances) than there are at present. Such a situation contradicts a basic tenet of the steady state theory, which holds that all large-scale properties of the universe, including the population of any subclass of objects like radio galaxies, must be constant in time.
- The second blow came in 1965 with the announcement of the discovery of the cosmic microwave background radiation. A direct prediction of Big Bang Model and the death knell for Steady State Theory.

5 Hints towards a hot beginning

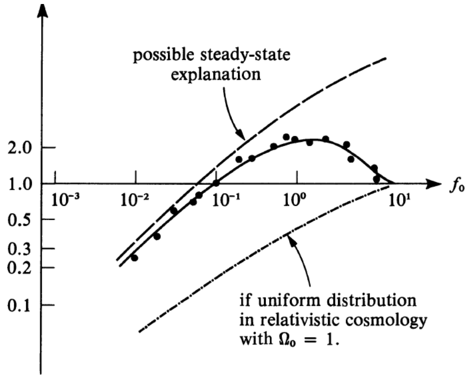


Figure 2: count of radio source brighter that f_0

$$N(f > f_0) = A{f_0}^{-\frac{3}{2}}$$

$$\frac{Number of QSO's}{Number of galaxies} = 10^{-\frac{5t}{t_0}}$$

Where QSO's refers to *Quasi-stellar object's* or **Active Galaxies**, now they are recognised as early galaxies in making. They spew huge amounts of X-rays and gamma rays as their central Super-massive BlackHole feeds on surrounding gas and matter.

In an expanding universe with the assumption of conserved mass-energy leads to preceding results.

Now similar to mass the radiation also gets diluted by D^3 but in addition as with stretching of D the λ also stretches by D which decreases energy and therefore dilutes mass density by D.

1. $\rho_m \propto D^{-3}$
2. $\rho_{rad} \propto D^{-4}$

These two combine to give:

$$\frac{\rho_{rad}}{\rho_m} \propto D^{-1}$$

**it is also possible to get these results from calculus.*
This predicted that there was an epoch when universe was radiation dominated (which corresponds to z≅1000) i.e. when universe was a 1000 times smaller.

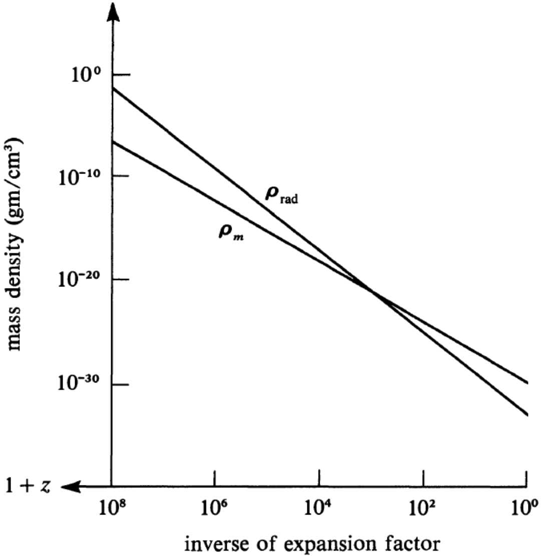


Figure 16.8. Evolution of the mass density of radiation, ρ_{rad}, and ordinary matter (baryons), ρ_m, as a function of the expansion factor of the universe. To obtain a timescale requires adopting a specific cosmological model, and this depends on how we treat the neutrino contribution.

The major evidence of **Big Bang** are: -

1. **Olber's Paradox**
   Although this is a quite old paradox it was popularized by Olber in 19^{th} century. This argues that if we assume infinite homogeneous Euclidean static universe then all line of sights end at a star surface which should cause the night sky to be as bright as an average star's surface.
   This is how we proceed:

   - If a star is at a distance R its brightness is $f = \frac{l}{4\pi R}$ where L is average luminosity but each this shell at distance R contains $n_0 4\pi R^2 dR$ stars this causes contribution of energy from that shell equal to $n_0 L dR$ which when integrated leads to $n_0 L R$ which increases without bound if we integrate to infinity.
   - More acceptable is that if the average radius is R then every line of sight intercepts a stellar surface if we have a homogeneous universe up to a distance $r = (n_0 \pi R^2)^{-1}$

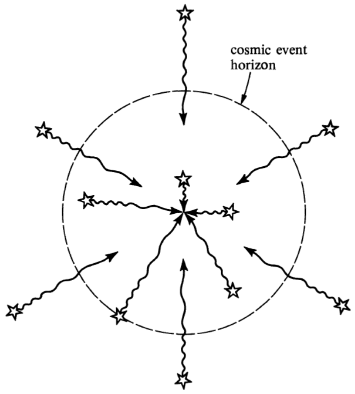


Figure 3: Resolution to the Olber's paradox

   The solution of this requires that universe is either not infinite and therefore had a beginning or is expanding or both.

2. **Cosmic Microwave Background Radiation(CMBR)**
   CMBR is a homogeneous and omni-directional glow of microwave radiation. It is the **"Echo of Creation"**. You can actually see this ancient light for yourself, when you set a random frequency on an old TV set, some of the random buzz that you see is this ancient ghastly relic from birth of the universe itself.
   CMBR actually looks like radiation from a body at a temperature of 2.7 K.

The actual story of discovery is quite marvelous and challenged only by the discovery of Hubble's constant. This starts with Gamow and his coworkers predicting thermal radiation from big bang remains. The calculation being further improved by Dicke and Peebles estimating it to be a black-body radiation of about 5 K. As a result, they started to make an antenna to detect this radiation however they were scooped by two Bell Labs scientist, Arno Penzias and Robert Wilson who had a found out an excess of 4.2 K antenna temperature which was isotropic and not of terrestrial, solar or galactic origin. When these two results were combined these two facts were known: -

- The radiation was indeed thermal origin of nearly 3K.
- The radiation was isotropic with a primary fluctuation due to motion of Earth around Sun and that of Sun in Milky Way Galaxy which once removed the radiation is very nearly isotropic with the only fluctuations expected from Quantum fluctuations in a very nearly uniform gas.

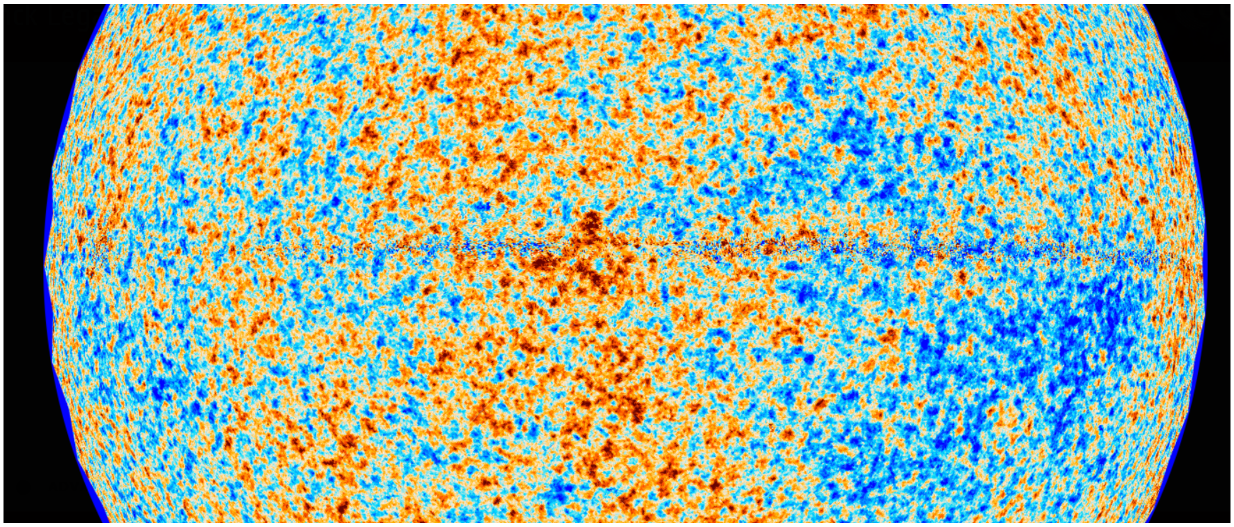
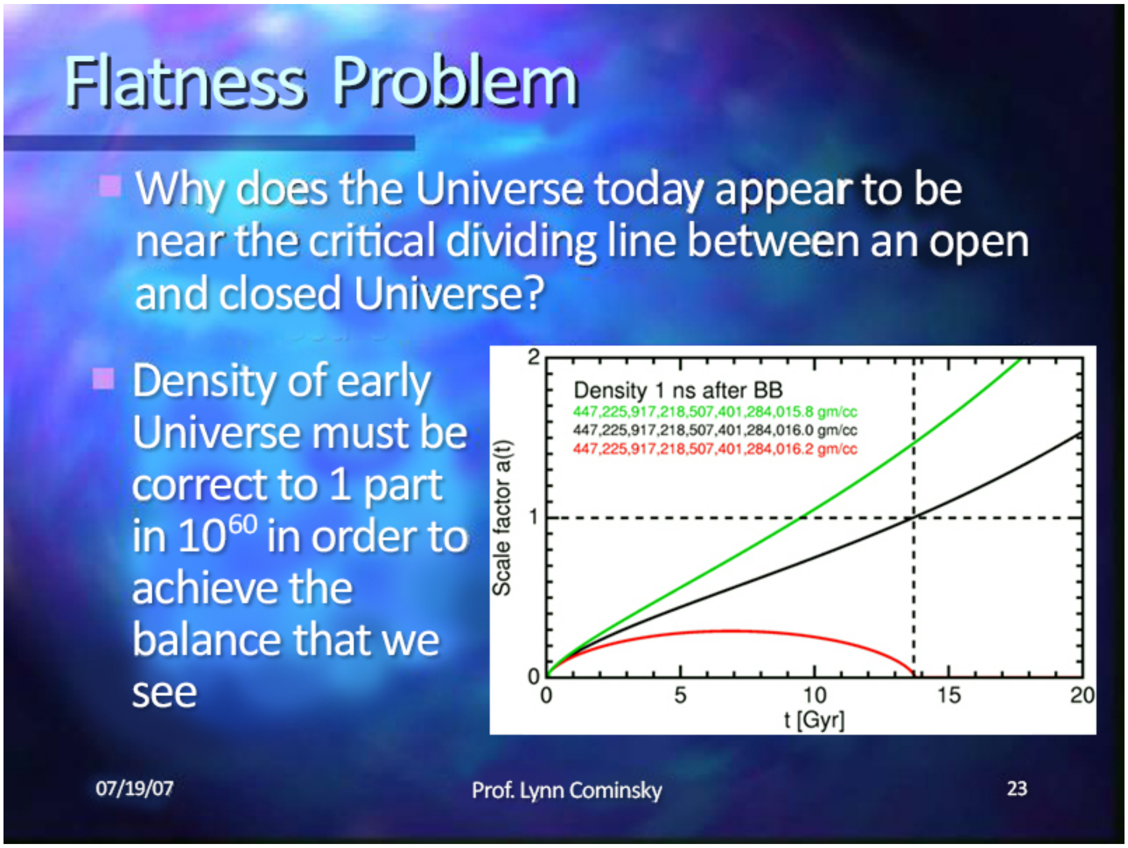


Figure 4: The Map of CMBR as captured by planck's satellite survey. (*credit: https://www.cosmos.esa.int/web/planck*)

↑ If there would have been some observer this is what the baby universe would have looked like .... just a lot, lot, lot brighter!!

The main flaws of BIG BANG theory are:

1. **Flatness Problem**
   A small deviation in density of universe causes a lot of difference to geometry of universe hence there are some physicists who argue how come universe is so finely tuned.



Flatness Problem

- Why does the Universe today appear to be near the critical dividing line between an open and closed Universe?
- Density of early Universe must be correct to 1 part in 10^{60} in order to achieve the balance that we see

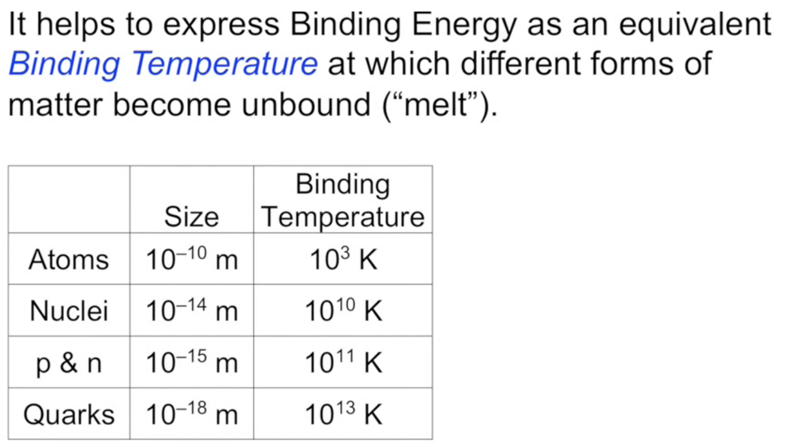
07/19/07 Prof. Lynn Cominsky 23

2. **Horizon Problem**
   It says that two points diametrically opposite with respect to us at epoch of CMB were nearly 28 Gly apart, but as the age of universe is only 14 Gyr how come they are at nearly same temperature as if in thermal equilibrium. Because information only travels at the speed of light, in order for these distant parts of the universe to communicate with each other, there has to be a faster than light speed transmission of information.

In answer to both the above problems- **Inflation** has been coined-out, which refers to exponential increase in space-time for a short time. during the very early stage of the universe, We will explore more about it in further portions.

6 The birth of a UNIVERSE

From the above discussions we are quite certain, that the universe began HOT, very HOT indeed, those were the temperature and densities so great that even heart of a supernova explosion seems chilly. The initial temperature of the universe was nearly infinite, but at those temperature and densities the laws of physics as we know them were not applicable. We have discovered that matter itself acts differently over temperature, now one might wonder if matter still consists of same everyday particles and temperature is just a measure of average velocity of the individual particles, then how do the laws of physics change with temperature!

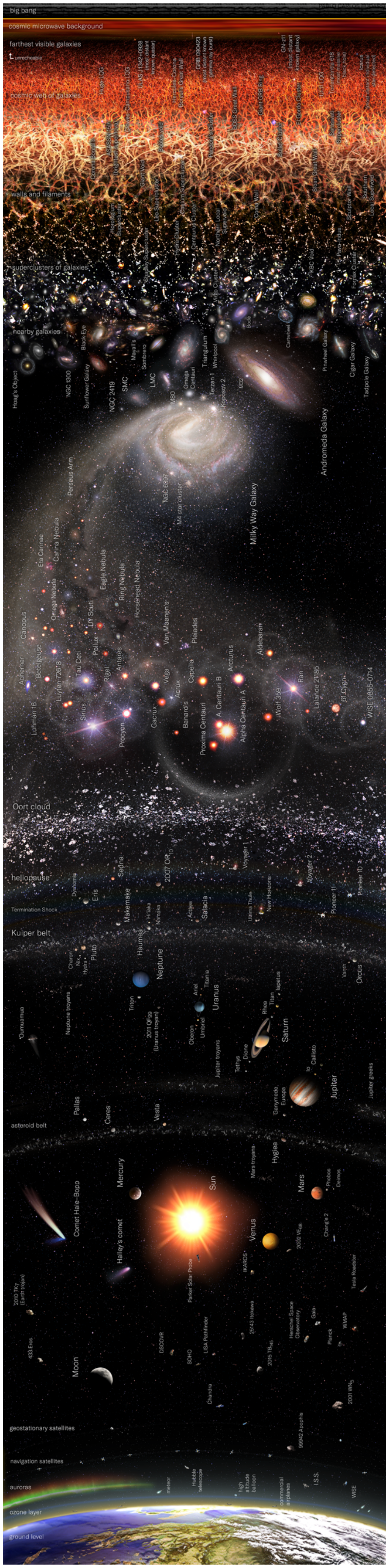


It helps to express Binding Energy as an equivalent *Binding Temperature* at which different forms of matter become unbound ("melt").

	Size	Binding Temperature
Atoms	10^{-10} m	10^{3} K
Nuclei	10^{-14} m	10^{10} K
p & n	10^{-15} m	10^{11} K
Quarks	10^{-18} m	10^{13} K

This has to do with the interaction time, the same particle, when colliding at 10k-20k temperature may have motions of its nuclear scale highly bound particles averaged out and thus immeasurable. While higher collision velocity (directly proportional to temperature) causes electronic phenomenon to be essentially too slow to show any practical effects thus here we study the effects of individual nucleons.

Then there is the mass energy equivalence dictated by theory of relativity – the higher the energy of colliding particles the more energy(total) they must create new particles. And the Heisenberg uncertainty principal also states that the smaller the time scales the higher energy virtual particles can be created (don't panic if you didn't know virtual particles were a thing), these virtual particles are the ones that mediate different forces in our universe. Thus, one can think that our universe has ranges of temperature where different forces dominate, and this means that the relative strength of forces changes with temperature. We observe that as the temperature increases electromagnetism and weak nuclear force subdues into one entity called the electroweak force. And now with the modern particle accelerators (LHC) that can go up to TEV's of energy we have discovered that strong force might also join the party.



← *This is a visualisation of how far different cosmological bodies are from earth, it follows a exponentially changing scale of length.*

(By Pablo Carlos Budassi - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=74584660)

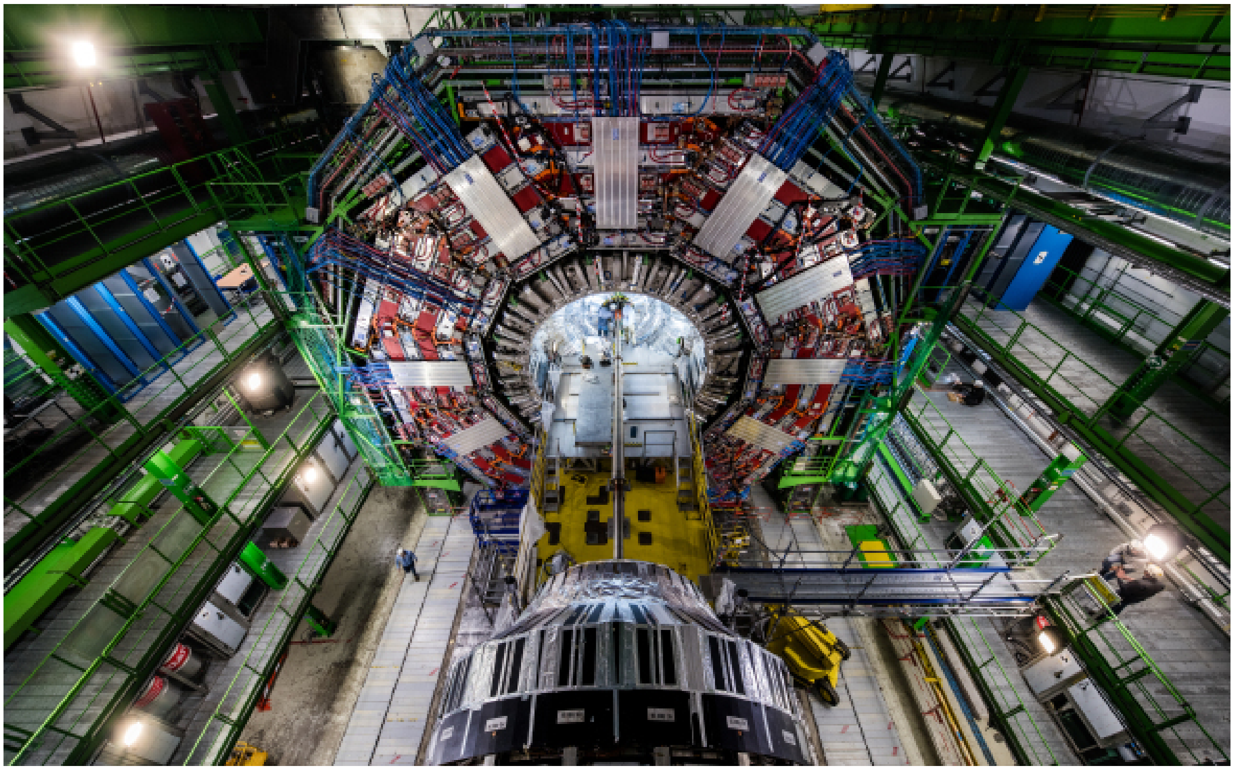
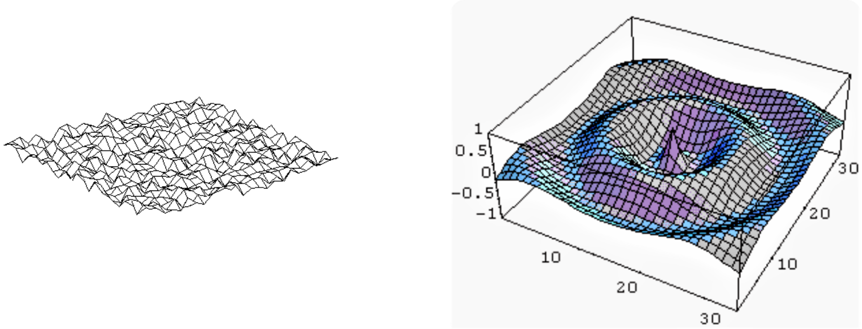


Figure 5: The compact Muon solenoid- CMS at CERN

But our particle accelerators are still insufficiently energetic to push the energy closer to the conditions that existed at the beginning and we are still unable to incorporate gravity into out unification. This is the problem that remains unanswered in Cosmology.

Quantum Field Theory(QFT): Learning about some QFT at this stage might be helpful, QFT or quantum field theory states that each particle that we observe has a associated quantum field, in fact the particle is considered as a local oscillation in it's quantum field, think of quantum fields as a drum skin and particles occur, when this drum skin is plucked. Different levels of excitation's of this quantum field results in different particles of different energies, like the excitation of electron field results in electrons but also in muons and tau. Here we run up on uncertainty principal, which dictates that each quantum parameter has a inherit uncertainty associated with it, in this case there is always an uncertainty in the energy of the field. Thus, Field can be momentarily exited to form particles. These particles are formed momentarily(These are the virtual particles, that we talked about earlier) but also obey the conservation principles (mass, charge, spin), they annihilate on the time scale set by the uncertainty principle.



(a) A quantum field at zero point energy

(b) An energised quantum Field

Figure 6: Quantum fields can be energised to produce particles

Normally we find a quantum field in the lowest energy state that it can be in, this is known as the zero point, or the quantum vacuum. If the field is locally exited from this quantum vacuum, it will rapidly de-excite producing particles. But it is also possible for a field to have complex relation of energy and state this might result in a sort of valley where a field can be stuck, having much higher energy. This is a state which is known as **The False Vacuum**, though the field has no particles in this state but still has a non-zero energy density. Since energy is equivalent to mass, this field exerts gravitational pull. But, contrary to popular opinion the gravitational effects exerted by a quantum field with a net positive energy density throughout the space causes a repulsive effect, it results in anti-gravity. In time we'll see how this positive energy density field might help to kick start the universe.

7 The Challenge:

To really understand the challenge at hand, we must first face what problems arise in our quest to understand the origins.

- Though incredibly successful, Einstein's General Relativity, is still an incomplete theory, and it comes into serious trouble when we try to apply it in the world of quantum mechanics. GTR is not quantifiable and has highly nonlinear algebraic form, while on the other hand quantum mechanics is a linear theory which has a probabilistic approach. Thus, both the theories are inherently incompatible in either the realm of mathematics or philosophy. To solve this situation physicist around the world are in a conquest of a "Theory of everything (TOE)" or "Grand unified theory". But to no luck we still don't have such a framework. Unification of quantum mechanics and relativity into a theory of quantum gravity still remains an open challenge.

- To test whether our ideas are correct, and scientist are not just- "throwing stones in a dark cave". We need to test them! Since we are testing for the events literally just fraction of seconds after the universe came into existence, we need to create similar situations, for this scientist use something whats called a Particle-Accelerator or Atom-smasher. We have built these machines throughout the world and have been building them from quite a long time now. The largest of these particle accelerators is at CERN collaboration, the Large Hadron Collider, its huge! With a tunnel diameter of 27 Km. But this is also not enough to recreate conditions just after the creation of the universe. With LHC we peer till billionth of seconds after the creation event. That's Good, but we need better. Thus, scientists are planning for an expansion to the previous LHC which will be even bigger, but this lies in the far future.

- One of the most obvious way to study the beginning is to look for it, the creation of the universe left some marks that- we can observe, and cleverly deduce, what is meant by them. But unfortunately, much of light from the distant galaxies that we observe (which are also the oldest galaxies) doesn't easily reach us. With the advent of this decade, humans are planning to put much larger telescopes in space – like the James Webb space telescope. With these much powerful instruments there is a likelihood that they might usher an Era of discovery once again.

8 The Chronology of Events

Here we present a step by step guide to the fiery beginning of the universe. Since our known scientific theories are incompatible at very small scale, we will start at 10^{-43} seconds, also known as plank epoch, this is the first time after the creation when the universe grew large enough that our laws of physics should start to make some logical predictions. What happened before that? The truth is .... We can't say- not to think we may never! But not now.

Our understanding has grown tremendously from the era when Galileo first took to the heavens, if you think it that way, we haven't done bad, and the quest still goes on (bolder than it has ever been) and we soon shall discover beyond any doubt or speculation- ***The mind of "god"***. *(Much of the information in this section is still being actively researched over, we want to paint a vivid picture about- what are the options that scientific community is considering. Most of the information here is bound to change, as we push further into the unknown...)*

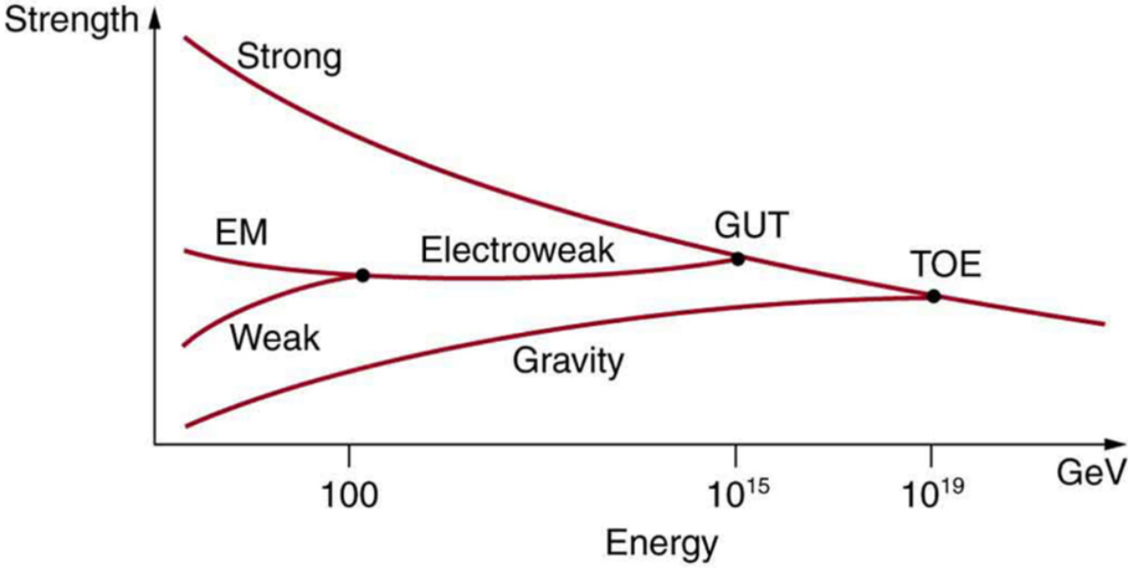


Figure 7: strength of different forces change with interaction energy.

credit: http://www.astronomycafe.net/FAQs/q1050x.html

- **The plank epoch($10^{-43}s$):** This is the time which is characterised by plank scale- this is the smallest scale where our current physical laws are correct, This epoch is characterised by temperatures of order $10^{33}K$. These orders of energies are far beyond the capabilities of modern human civilisation to achieve, nothing can be said what happened during or before this epoch. The Universe was essentially microscopic and quantum mechanical effects of gravity ravaged.
- **The Grand Unification epoch(10^{-36}s):** This is the time when symmetry between the laws of physics broke, gravity separated from what is still electrostrong force *a.k.a* Grand unified Force. This is the time when Matter dominated over antimatter as a result of broken symmetries, we shall talk about it later in the chapter. In this epoch, the universe was populated by Quarks and anti-Quarks it is thought that during this time for every 10^9 anti-quarks, $1 + 10^9$ quarks existed.
- **Inflationary epoch(10^{-32})s:** This is the time in the History of universe when universe undergoes tremendously Rapid expansion known as **INFLATION**, This is the bang of the Big Bang. But why do we need Inflation?

8.1 INFLATION:

Here we discuss Inflation, by this time you would have heard about it many times and might be wondering- what the heck is it!

The old Big Bang theory had tonne of problems- What drove the universal Expansion in the first place? What kept the universe expanding even if the gravity was so strong that it should have collapsed back? What makes the universe so smooth on the largest of scale? (*If you were to look in the sky and see two diametrically opposite point on it they are essentially too far apart for light to ever travel but still you see they look essentially the same- even the cosmic background radiation from both the points has the same temperature*),
Why is the Universe "FLAT" on the largest of scales? - here we mean 4-dimensional flatness in perspective of General Theory of Relativity!
If the universe evolved to this size - from a size much smaller than a proton, then any fluctuation, any over dense region should have been amplified tremendously during an expansion.

To solve all of these problems (and few others) Alan Guth proposed the idea of Inflation - inflation proposes that universe had to essentially double in size over every $10^{-34}s$, and the universe probably went through a 100 or more of these doubling periods. This effectively amounts to any spot the size of proton growing to 10^{19}*light-years* in size. This is hopelessly huge, tremendous, gigantic, words do fail to describe this valour of early universe.

Some words of caution!

→ Though this looks like a faster than light expansion, one might say that its not possible as Einsteins GTR forbids FTL. but you would be wrong in this case, as this is the faster than light movement of space itself, which is not forbidden.

→ People often think of Big Bang as an explosion of some sort(a very large one indeed), but this is a wrong image, as the Big Bang represents creation of universe itself, there is really no point outside universe where an observer can stand and look back at an expanding universe- Big Bang occurred at all points at once, since all points were really created in this event.

→ Did time begin at Big Bang! This is still a doubtful question and our theories can't answer it. We aren't even sure if this is a valid question as, we still struggle to define - What is Time?

What caused it: The universe had particle's after the Grand unification epoch(mostly quarks, gluons and leptons), but it is also thought that it had a quantum field carrying a very high energy density. This might be the Higgs field, or a hypothetical field called the **Inflaton field**. This inflaton field by the virtue of its very-very energy density exerted a positive pressure over space. As discussed earlier this field exerts a strong anti-gravitational effect, this is what leads to exponential expansion. The quirk with this inflaton field is that **as more space is created the field strength does not drops**. inflation is what does the magic and flattens out the universe- it dilutes the universe with the same mass energy content.

How did it stopped: Inflation started at about $10^{-36}s$ after the beginning. and ended at $10^{-32}s$. The inflaton field was pumping up the expansion of the universe, but in doing so it was also cooling the universe, the universe gets super-chilled from $10^{27}K$ to $10^{22}K$ that's a decrease of 100,000 times. though you might not want to make a ice-cream at those temperatures. This causes the field to come in a meta-stable state it has much higher energy content than its temperature. It's like super cooling water below freezing point, it doesn't becomes ice, but as soon as it is tinkered all of it turns to ice at once. This is what happened with the inflaton field, as it is a quantum mechanical field, it is also ruled by the ultimate uncertainty. Once any point in this exponentially expanding universe is bumped because quantum fluctuations, it stops accelerating, and bumps its neighboring points too so, they too stop inflating. This ever increasing wave travels at the speed of light, and all of the parts within find them as having lower value of inflaton field. but for a field to decrease its energy it must transfer its energy to something else. Inflaton field dumped all of this energy into particles- Inflatons. Vast amounts of inflatons are created. These decay via various processes into other particles- mostly protons,neutrons,electrons and photons. The particles we now have and love. This creation of particles re-energises the universe to temperatures before the inflationary period. The energy density of the universe before inflation does not affects the one after inflation as it was essentially negligible before.

By the end of inflation, the universe was much bigger and had a lot more particles, it had been flattened to a very high degree and was expanding still- but with a very tiny acceleration(because inflaton field did not fell all the way to zero, it happened to have some intrinsic value above zero, this acceleration effect we now attribute to dark energy!), the speed of expansion maintained the value it had gained at the end of inflation, thus it was the inflation, which breathed into the universe- the "bang" of the big bang.

- **Quark Plasma epoch($inflation - 10^{-12}s$):** The universe has cooled enough for the strong force to separate from the electro-weak force, but the temperatures are so high that the quarks cannot combine into hadrons thus the universe at this stage is soup of quarks, anti-quarks, gluons, and photons. there are slightly more quarks than anti-quarks as dictated in the GUT epoch. These are the energies where our particle accelerators like LHC and RHIC can reach.

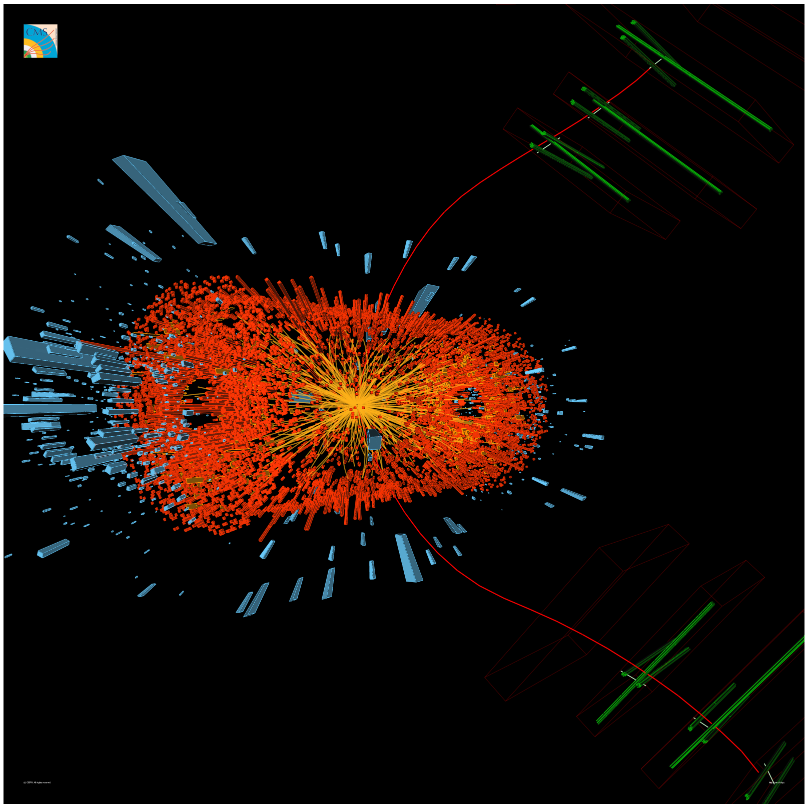
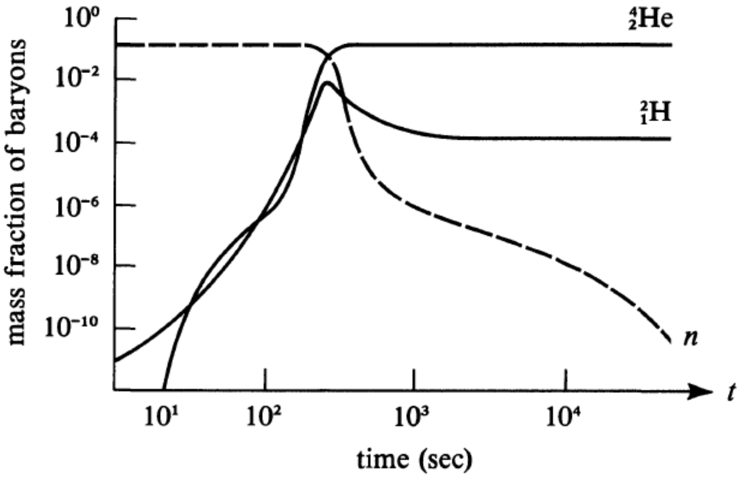


Figure 8: A collision event at CMS, CERN. showing quark-gluon plasma.

credit: https://cms.cern/news/new-cms-heavy-ion-results-quark-matter-2012-conference

- **Hadron Epoch($10^{-7}s$):** At 10^{13} K and $7\times10^{-7}s$ seconds into history, the universe drops below the threshold for protons and neutrons by a process called Baryogenesis. As a result these particles stop being major constituents of the universe. There is a general annihilation of p with anti-p and n with anti-n, but the universe is left with a small excess of matter over antimatter. Before the annihilation the number of protons and neutrons were kept equal to those of the photons by equilibrium pair production events. By the end of this epoch there is a 1:7 ratio in number of neutrons and protons.
- **Neutrino decoupling ($1s$):** By the time one second has passed most of the neutrons and protons we now observe and love, have already been created. By this time universe expands and falls below the threshold energy of W and Z particles, as a result these particles are now no longer created freely and being the primary mode of interaction of neutrinos with ordinary matter, neutrinos decouple from ordinary matter and are free to fly freely. It is also believed that slight over-densities in this period might result in formation of primordial black holes. We might appreciate that till now only one second has passed since the creation of universe itself and our part of the universe that we now see today (*observable Universe*) is only a sphere 10 Light-years across.
- **Lepton epoch($10s$):** The universe has cooled enough for hadron/ anti-hadron equilibrium to condense but leptons(*e.g.*-electrons) still remain in equilibrium with radiation(photons).
- **Big Bang Nucleosynthesis ($10 \rightarrow 10^3$s:)** The Temperature dropped to the threshold where neutrons and protons can combine and form bare nucleus under the effect of strong force, this lead to the initial spurt of light element creation like- Helium, Deuterium, Lithium.
  During nucleosynthesis all the free neutrons are swept up and bound into nuclei. Once bound in this way, the strong interaction between the protons and neutrons stabilizes the neutrons preventing them from decaying. By the time that nucleosynthesis starts, the decay of free neutrons has shifted the p/n ratio to 87/13. Out of every 200 particles 26 neutrons will combine with 26 protons to form 13 helium nuclei. This leaves us with 26% Helium concentration which is well established from observation of very old star populations. The graph below shows how concentrations of different elements changed overtime the early history of the universe.



- **Radiation dominated Universe ($10s \rightarrow 370,000 years$):** This is the first long(*according to us!*) period in the violent history of the universe, during this epoch the matter concentration remains more or less fixed, There are a lot more photons(particles of light) than the amount of matter particles in the universe. Thus, we say that this is a radiation dominated universe. This is the time when universe will undergo- Baryon acoustic oscillations. Which are essentially colossal sound waves moving at half the speed of light through dense ambient plasma environment, These start because the rapid inflation caused very small quantum mechanical fluctuations to grow to tremendous size, creating over-dense region's, fueled by outward photon pressure and inward gravitational pull of - ***Dark Matter***. This will result in very peculiar web like distribution of matter, which we observe today as the large scale structure of galaxy clusters and super-clusters. During this Epoch universe exists as a very hot dense plasma. One of the properties of plasma is that, it scatters photons. Thus once a photon is created it doesn't goes very far away before colliding with another charged particle. Thus, the universe is an opaque, soup of very high energy plasma and photons.

- **Recombination** $(370,000yr)$**:** This was the time when universe cooled down, to the point where neutral atoms can form. The Opaque plasma gave way to clear unionised gas- the universe became transparent, for the first time, the photons could travel substantial distance before being absorbed. The radiation that was set free at this time was very energetic ultraviolet and gamma rays. This was the first light in the universe.

- **Cosmic Dark Ages** $(370000yrs \rightarrow 150000000yrs)$**:** During all this time the universe was expanding, gravity was bringing together large lumps of pristine hydrogen gas, and Dark matter was forming large scale filaments, all of this took time. No star or any other source of light existed during this time. The universe was dark.
  The light that was released at the time of recombination travelled vast distances, in an expanding cosmos. With the universal expansion the wavelength of photons were also expanded (they were red-shifted).
  There is a particular wavelength (656.281 nm) which is absorbed readily by atomic hydrogen, this wavelength is called Hydrogen-α or H-α wavelength. As the glow of recombination red-shifts, one by one different energy photons are pushed to this wavelength, and Thus, into the danger zone! they get absorbed by all that pristine molecular hydrogen gas that is present is vast amounts.

- **Stelliferous Era** $(150000000yr \rightarrow present)$**:**

The end of the dark-ages gave way to the first stars. These first stars were massive and short lived, these spewed vast amounts of energy into the surrounding gas, causing it to re-ionise. By this time the universe was tenuous enough that plasma didn't scattered much of light. By analysing the last H-α wavelength that was absorbed by the hydrogen gas, we come to know - when were the first stars born. These photons from the age of recombination that survive being absorbed by hydrogen, are what constitute the Cosmic Microwave Background Radiation (CMBR), what we talked about earlier. When this radiation was emitted it was in the form of highly energetic gamma rays, but travelling through an expanding universe this Radiation has now Red-shifted into weak radio waves.
The First generation stars spelt the first heavy metals- iron, gold, uranium, etc. Inside their cores and in their spectacular deaths. And seeded the surrounding gas with these elements, which will someday find themselves inside our bodies. This is the Stelliferous Era, where the stars rule the night, and the universe is at its greatest splendour. And not to mention there are some bunch of intelligent Apes who love to gaze out into the sky, and ask - ***"HOW DID IT ALL BEGIN!"***.

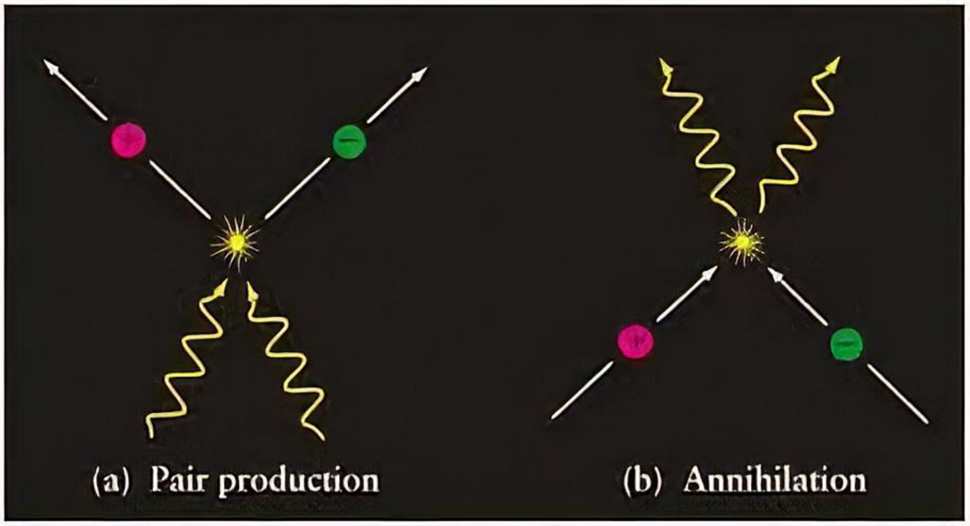


Figure 9: Matter anti-Matter interaction.

9 WHY IS THE UNIVERSE - ”MATTER”

9.1 Baryon Asymmetry Problem

The world around is made entirely out of matter. Otherwise, when matter and antimatter interact, they would annihilate each other by emitting radiation. This radiation emitted would be enormous, and of characteristic frequency. If there are such interactions in the observable universe, we would have witnessed some evidence for such a phenomenon. Thus, it would be natural to assume that the universe consists almost exclusively of protons, neutrons and electrons.

But, an extremely attractive idea for the creation for the origin of mass-energy in the universe is its creation from the quantum vacuum in the violent early expansion of space-time from the Big Bang. Thus, in very early instances, the number of particles and antiparticles were by definition, zero. If particles and antiparticles were subsequently pulled out of vacuum, the resulting universe would have an equal number of particles and antiparticles. But this is in direct contradiction with our observations. Moreover, the matter and antimatter produced would eventually annihilate each other leading to a universe filled with radiation.

From our observations, we are compelled to say that the universe started out more matter when compared to antimatter. In fact, in the present-day world, we can roughly estimate that there are about a billion times more photons than there are particles of matter. From this, we can say that one in a billion particles of matter survived from this annihilation.

Before delving into this, let us take a step back and have a look at various symmetries which we thought were fundamental.

9.2 Symmetries

The 3 main symmetries scientists thought were fundamental are: Charge symmetry, Parity Symmetry and Time Symmetry or, in short, C, P and T symmetry.

- **C-symmetry:** : It is a transformation that switches all particles with their corresponding antiparticles, and thus changes the sign of all charges. We can see this in our surroundings with both electromagnetism and Gravitation following the same.
- **P-symmetry:** The idea behind this is that equations in particle physics are invariant under mirror transformations. Thus, it was predicted that the mirror image of a reaction occurs at the same rate as the original reaction.
- **T-symmetry:** This is theoretical symmetry of physical laws under the transformation of time reversal.

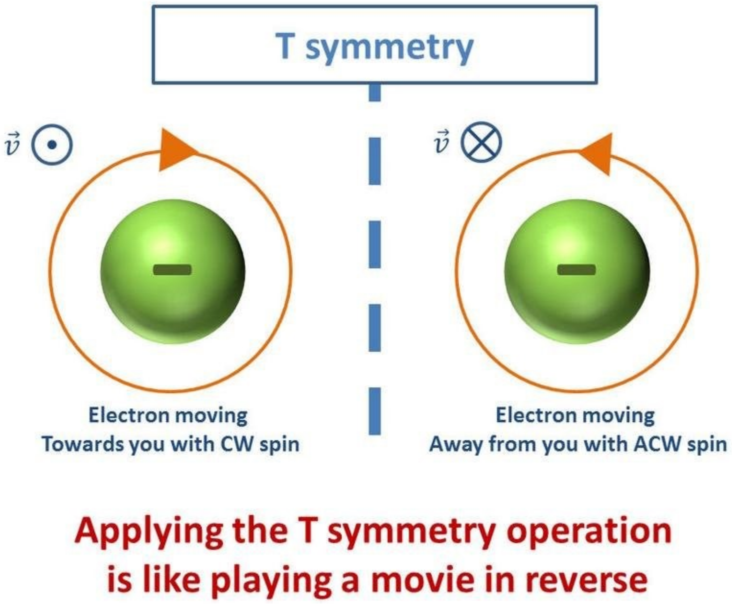


Figure 10: Link for a gif regarding the same- https://media.giphy.com/media/SVr9MZx7UHnVm8VT7z/source.mov

This seems blatantly false as it violates the Second Law of Thermodynamics.
However, the second law of thermodynamics is applied only to macroscopic systems. So, is T-symmetry valid for a microscopic physical particle?
In due course of time, it was realised that C, P and T symmetry does not hold true even for microscopic particles. But then, there was a possibility that a combination like CP symmetry might hold true, but later, even this was violated. To date, only CPT symmetry has stood the test of time, or rather, we haven't found any violations of the same.

Violations:

- **Parity Violation:** In 1956, Chien-Shiung Wu her team were studying the decay of cobalt-60, an unstable isotope of the element cobalt. They cooled the atoms to about $\frac{3}{1000}^{\circ}$ above absolute 0.
  Then, they applied a strong magnetic field to orient all the nuclei so that their spin points in the same direction. Cobalt-60 is unstable and decays into Nickel-60 with the release of an electron and an anti-neutrino.
  Now, if parity symmetry is to not be violated, we should observe the direction of the electrons to be in the direction of the spin or, opposite to the direction of spin in equal probability.
  Consider, a mirror placed perpendicular to the direction of spin of the nuclei, say the z-axis. Now, in the mirror world, we observe that the direction of spin remains the same while the z-axis inverts.

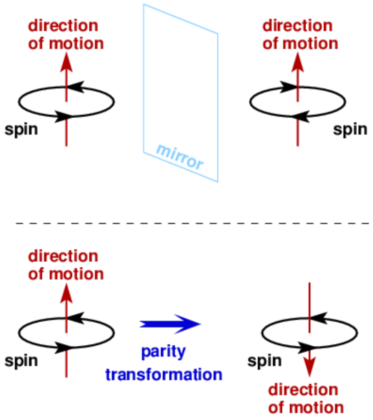


Figure 11: A GIF regarding the same

Now, if the nuclei emit more electrons in a preferred direction, say in the positive z-direction. The mirror image of the nuclei will more emit electrons in the negative z-direction and vice-versa. Hence, we see that there should be an equal number of electrons being emitted in both directions.
However, electrons were emitted preferentially in the direction opposite to nuclear spin (negative z) which violated the parity symmetry.
Hence, in essence, you could technically figure out if you are in mirror world or not by observing the direction of emitted electrons in this experiment.

- **Charge-Parity Violation:** In 1980, it was shown that even CP symmetry does not hold by observing the decomposition of a neutral K meson.
  Hence, if P-symmetry does not hold and CP- symmetry does not hold, we see that even C-symmetry would not hold. Also, we haven't found any violations of CPT symmetry. Therefore, we can assume that even T-symmetry would not hold as CP- symmetry is violated while CPT symmetry holds true.
  We can imagine that CP violation some how "undoes" the effect of T-violation.

- **Time symmetry Violation:** Although it was theorized earlier, later on, scientists found particles that directly violated T-symmetry. When a pair of quarks are held together by the strong force, there are 2 different kinds of arrangements and they can switch back and forth via the weak force.
  It was observed that switching in one direction takes longer than switching back.
  This violates T-symmetry as otherwise the switches would have taken equivalent time.
  Hence, after all this we can only assume that only CPT symmetry truly holds true. If CPT symmetry somehow fails, we would have to rewrite a lot of physics of the last century including special relativity.

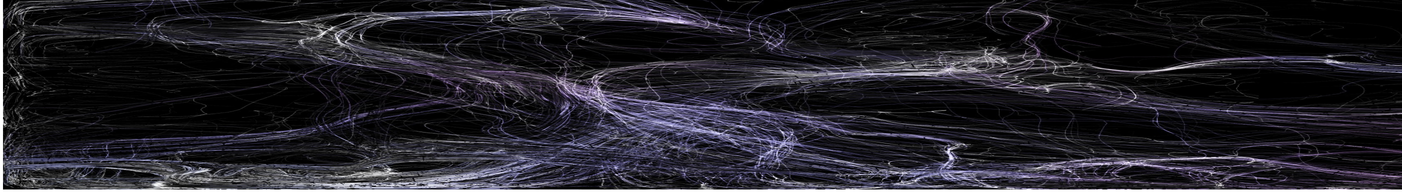
Now finally, let us come back to the problem at hand i.e. the matter antimatter asymmetry. Instead of asking the "why" let us first ask the "how". Trivially, there should be a baryon number violation to produce an excess of baryons over anti-baryons. Baryon number can be defined as the difference between the number of baryons and anti-baryons in a system. Along with this, C-symmetry would have to be violated as interactions would have to produce more baryons when compared to anti-baryons. Also, CP symmetry would have to be violated as otherwise there would be an equal number of left-handed baryons and right-handed anti-baryons and vice-versa. We have observed C-violation and CP- violation but have not observed particle interactions where conservation of baryon number is broken. Hence, even today, there is no consensus on the origin of matter and it remains as one of the greatest mysteries ever.

9.3 Other Theories:

→ What if there is actually an equal number of particles and anti-particles but we by chance lie in an antimatter poor part of the universe ?

→ What if the tables are turned beyond our observable universe? i.e. there is more antimatter when compared to matter beyond our observable universe.

→ Well, the universe we observe is mostly uniform in the large scale (courtesy of the CMB), so it would be a stretch to say that it would this anisotropic outside our observable universe. So, the chance of this is quite unlikely.



Clusters of Galaxies and the Expansion of the Universe

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A Basic Overview

So after a short intro on the fiery beginnings of the *Universe*, we head on to understanding very interesting concepts and phenomenons from the book – **The Physical Universe : *Frank H. Shu***. Though the topic we are going to see is self-explanatory in its title, it is good to break it down.
We would see clusters of galaxies, be it only 2 membered(*binary*) or multi-system(*millions or billions!*). The galaxy clusters have very peculiar features, some of which could be explained, yet some remain questionable till date. Followed by this, we would try to find answers behind the expansion of the universe, the ***Hubble's Law*** and its interpretation. And **don't worry at all**, since we have tried to explain in the easiest way possible! So without any further ado, let's ride along......

1 Interacting Binary Galaxies

Galaxies were generally discussed as isolated entities, free from the influence of other galaxies. In reality though, just as there are interacting binary stars ,there are interacting binary galaxies observed as well. Strongly interacting pairs of galaxies constitute a very small percentage of the total number of galaxies known to us .Spectacular examples of rings, bridges and tails have been catalogued by *Vorontsov-Velyaminov* and by *Arp*. If two galaxies have very different masses, then the **smaller one can pull out material from the near side of the larger galaxy** into a **bridge** that temporarily spans the gulf between the two galaxies .

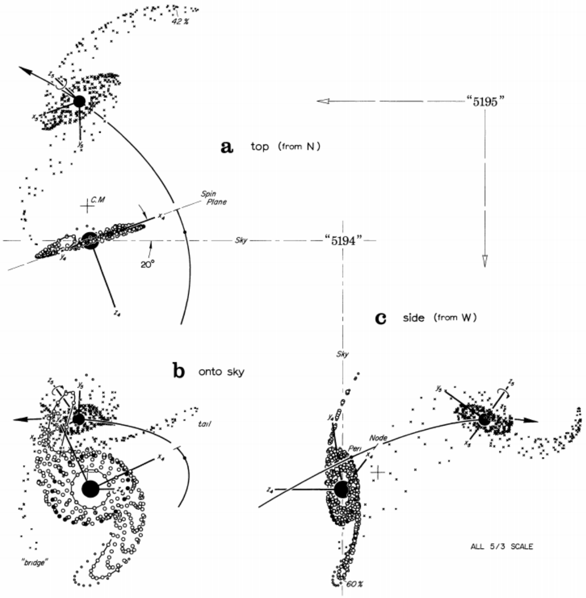
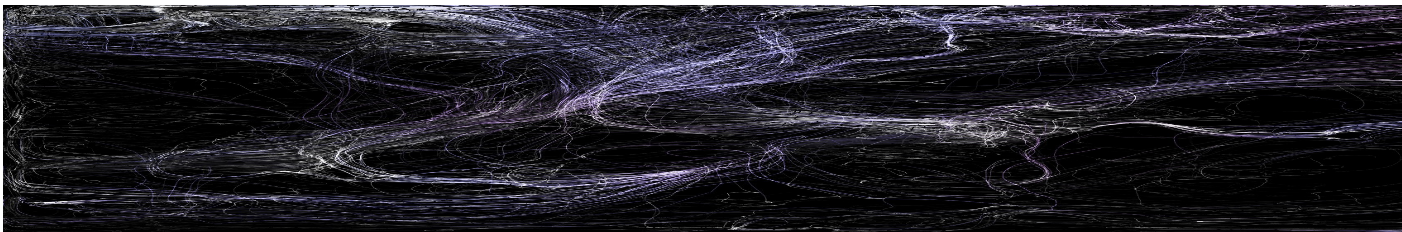


Figure 1: *The above figure shows a numerical simulation of a bridge-producing encounter . A direct close flyby of a small galaxy past a big disk galaxy draws out a bridge of material that temporarily spans the gulf.*



If the encountering galaxies have nearly equal masses , then a tail from each galaxy may develop that generally extends away from the main bodies .This non-intuitive result arises as the tidal interaction is much stronger as compared to the *familiar example of the Earth-Moon system*.

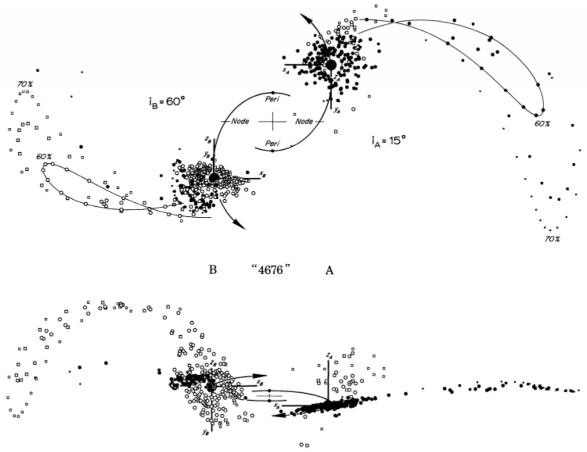


Figure 2: *The above figure shows how a direct flyby of two disk galaxies of comparable masses yield two tails that extend away from the main bodies.*

Long bridges and tails are best produced when the orbit of two galaxies are bound, so that the two systems are not flying past one another too fast when they reach the closest approach. In very rare cases, when galaxies interpenetrate, exotic looking **ring galaxies** can be formed .

2 Mergers

Close encounters between galaxies excite much internal motion in them. The energy to produce these motions must come from the orbital motion. In a gravitating points (stars) system, it disperses as radiation. The collection of stars would tend to conserve its total energy. The stars might transform orbital energy into random motions.

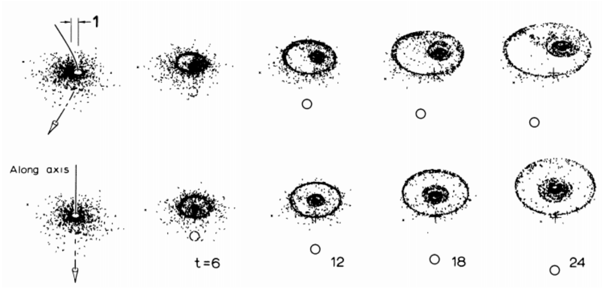


Figure 3: *Two numerical simulations of ring-producing encounters. An interpenetration of a disk galaxy by another massive body yields rippling waves of rings*

Indeed, it can be argued on statistical grounds that repeat encounters between two bound galaxies must tend to bring the two galaxies closer together, consistent with the constraints of the **conservation of total angular momentum and total energy**. This is based on the *second law of thermodynamics*: the encounters increase the entropy of the universe.

2.1 *Merger Process*

The close encounter of two bound spiral galaxies,must produce a merger into a single pole of stars. This merger process involves a form of ***violent relaxation*** in which violently changing gravitational fields help to produce a final smooth distribution of stars. Such a pile of stars would probably strongly resemble an *elliptical galaxy*.

2.2 *Alar Toomre's argument*

His argument is beguiling,and proceeds as follows. Of the 4000 or so NGC galaxies, perhaps a dozen are interacting systems exhibiting **spectacular bridges** or **tails**. On the other hand, the numerical simulations show that such geometric forms are transient phenomena that cannot be maintained for more than a few times 10^8 years.

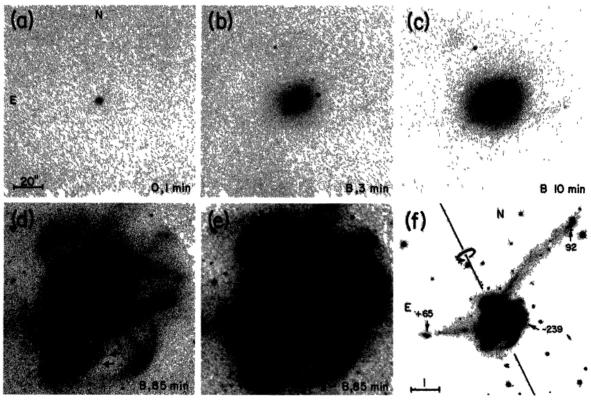


Figure 4: *Successfully deeper photographs of NGC7252 reveals complex set of filaments surrounding a central body that resembles a giant elliptical galaxy.*

Question 1

Could there be any flaws in his argument? Think and explore.

3 Hierarchical Clustering

Well, well ,well ...So previously we have seen double galaxies, but they do exist in the form of small rich clusters as well. In the Local Group (*a group of galaxies*) like the one containing our **Milky Way** and **Andromeda**, there may be dozens as well members extending over a radius of million light years. This can even exist on a further larger scale, extending to hundred of million light years, earning the name Super-cluster. These have been confirmed to exist by Holmberg, Reiz and de Vaucoulers, yet whether clusters of super-clusters exist is questionable.

3.1 *Rich Clusters of Galaxy*

The most closest rich cluster to us is the **Virgo cluster**. It boasts of a collection of 200 bright galaxies (68% spiral : 19% ellipticals : rest irregular/unclassified), and one of the brightest galaxy being Messier-87(*M-87*), an elliptical galaxy. Another notable one is *Coma cluster*, roughly 7 times as far as Virgo.

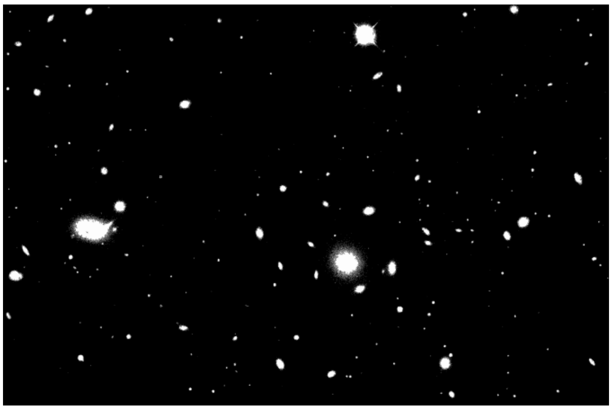


Figure 5: *Central region of Coma cluster containing two super-giant ellipticals,which have grown bloated.*

These rich clusters share some **common features** :

- **Scarcity** in the number of Spiral galaxies.
- The presence of one or two very luminous supergiant elliptical galaxies near the center of cluster. This supergiant elliptical galaxy is known as **cD galaxy**, for some historical reasons which the author finds "*not particularly illuminating*". As a result of their size, they dominate the appearance of the cluster.

To know why these common features come to the picture, let us head to the next sub-sections.

3.2 *Galactic Cannibalism*

A cD galaxy is seen to have possession of envelope of stars, like a bulge. An example is NGC6166, a radio galaxy residing in Abell cluster 2199. Oemler noted in his surface photometry on these extended envelopes of cD galaxies that the rate of dropping of brightness from the center is slower than given by **de Vaucoulers law**, which applies to ordinary galaxies.

$$\ln I(R) = \ln I_0 - kR^{1/4} \tag{1}$$

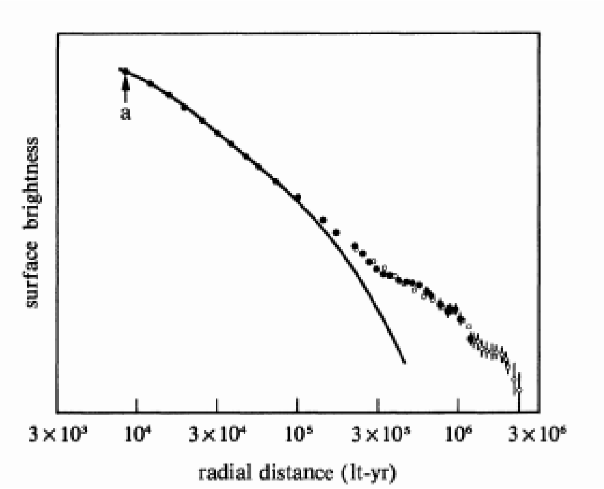


Figure 6: *Deviation of the law for cD Galaxy.*

Hence it was proposed that these cD galaxies grow bloated by cannibalising(*yes...like the old-age man cannibals*) on smaller neighbours. This is explained by two phenomenons :

- **tidal stripping**, and
- **dynamical friction**

3.2.1 *Tidal Stripping*

This concept is commonly known to us, as we hear about tides in our common life. The principle is based on ***Roche's Limit***, that is, an object of mass m and radius R, held together by self-gravitation and which approaches within distance r of massive body M, will be ripped apart by tidal forces at the critical limit of

$$r = (\frac{2M}{m})^{1/3} R \tag{2}$$

A quantitative visualisation can be that a 500 times heavier cD galaxy can rip apart a galaxy stars at r = 10R. Not all shredded material can be captured gravitationally, some of the stars may enter the orbit about cluster as a whole.

> **Demystifying the Equation:** Consider a mass m body of radius R placed in a circular orbit of radius r with massive body M at the centre. The gravitational forces at the extreme ends of the body m would be $\frac{GMm}{(r-R)^2}$ and $\frac{GMm}{(r+R)^2}$.
>
> The difference between the forces at ends can be calculated to be found as, under the assumption that $R \ll r$
>
> $$\frac{GMm}{(r-R)^2} - \frac{GMm}{(r+R)^2} = \frac{2GMmRr}{r^2 - R{2}^2} \approx \frac{2GMmR}{r^3} \tag{3}$$
>
> Stars at extreme ends of m experience a force per unit mass of
>
> $$\frac{Gm}{R^2} \tag{4}$$
>
> If disruptive acceleration of $\frac{2GMR}{r^3}$ exceeds $\frac{Gm}{R^2}$, galaxy m will be ripped apart. Hence equating both the forces would given the limit r, and hence the above equation(2).

But there is **an issue**. Galaxies do not have uniform distribution, and the density is largest at center. The rarified outer portions will be exceeding Roche's Limit and tidal stripping would effectively occur. But the dense cores will have fall quite deep into the heart of cD galaxy to have some effect. Since this deep encounter is rare, a cD Galaxy has to do something else to gobble the cores of other galaxies, which it does by *Dynamical Friction*.

3.2.2 *Dynamical Friction*

Cause of its **origin** can be easily understood. As heavy dense core m moves through a medium of stars, the core deflects the stars on it way. These deflections kind of pile up behind m, and this extra mass starts pulling the mass m, and reduces it velocity relative to the stars. This net effect of dynamical friction ultimately brings the galactic core to center of a cD galaxy. And now......its *DINNER TIME*!

The core of the galaxy is highly massive, hence the defections are correspondingly greater. Quantitatively, the dynamic friction can bring the core of galaxy near the center of a cD Galaxy in terms short compared to 10^{10} years, the age of the universe.

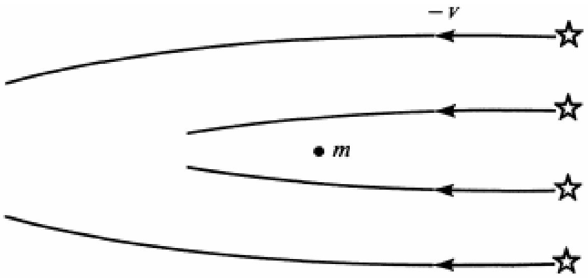


Figure 7: *Dynamical friction arises when a mass m moves at a speed wrt stars distribution which are statistically at rest.*

The Relaxation Time (t_{relax}): Just like we kiddos studied in a resistance wire, a rough estimate of the relaxation time for stellar encounters can be estimated. Considering all stars to be a sphere of radius r, if any star were to enter this sphere of influence, it could suffer an encounter. And hence t_{relax} is the **time between two successive encounters**. If star density is n, velocity of star V, then we want the cylindrical volume of star's sphere of influence swept in one t_{relax} to contain one star. That is

$$n(\pi r^2 V t_{relax}) = 1 \tag{5}$$

A *sensible choice* of r would be at which the gravitational PE of pair of stars equals a typical KE of a star.

$$\frac{Gm^2}{r} = \frac{mV^2}{2} \tag{6}$$

Thus t_{relax} comes out to be $t_{relax} = \frac{V^3}{4\pi G^2 m^2 n}$. But deep analysis shows that many smaller deflections are much efficient than one big deflection, by a factor of $\ln(\frac{2R}{r})$, R being radius of cluster-core. Hence the equation comes out to be

$$t_{relax} = \frac{V^3}{4\pi G^2 m^2 n \ln(\frac{2R}{r})} \tag{7}$$

Now **back to our case**, taking mass m of galaxy, mass of each scattered star to be m_*, N stars in volume $\frac{4\pi R^3}{3}$. Also $Nm_* = M$, mass of the cD galaxy. So the equation simplifies to:

$$t_{relax} = \frac{(rV)^3}{3G^2 Mmn \ln(\frac{rV^2}{Gm})} \tag{8}$$

Question 2

Check whether big galaxies would be cannibalized in preference to smaller ones.

3.3 *Hot Gas in Rich Clusters*

Satellite observations in early 70s showed that X-rays pour from spaces between galaxies in rich clusters. It was concluded that this occurred due to presence of *hot gases with temperatures between* 10 *and* 100 *million degrees Kelvin*. The mass of these gas was nearly **comparable** to luminous part of the clusters. The **origin** of this gas was debatable – earlier it was believed that the gas may have come in clusters from intergalactic space. But the analysis of gas showed presence of elements like iron, which is synthesised in deep core of the massive stars. Hence it is believed to come from within the galaxy.

Ideas and Hypothesis

- Mathew and Baker proposed that the interstellar medium might be so hot in the elliptical galaxies that gas lost from stars would continuously blow out of the galaxies, like *solar wind carries material away from corona of the Sun*. This would work for both isolated elliptical as well as those in the clusters. In **spiral galaxies**, there is too much gas in the disk to maintain such a high temperature, so it was hypothesised that wind mechanism is **inefficient** here for removing gas from disk systems.
- A more efficient method **ram pressure** was also proposed, which was created by relative motion of galaxies through cluster medium. *This is similar to how wind knocks off the hat of fast-pedaling cyclist.*

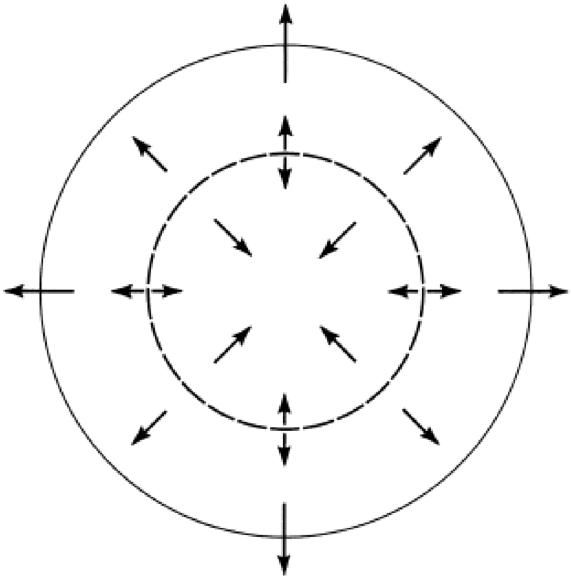


Figure 8: *Gas expelled from stars in main body of elliptical galaxy may heat up and blow outward in galactic wind, or flow inward toward galactic nucleus.*

3.4 *Missing Mass in Rich Clusters*

Shane and Wirtanen and de Vaucouleurs formulated that the number of galaxies per unit area of Coma cluster varied with distance r from center according to

$$\eta(r) = \eta(0) exp[-(\frac{r}{r_0})^{1/4}], \tag{9}$$

$\eta(0)$ and r_0 being constants.

Figure 9: *Plot of the equation(9).*(Plotted via matplotlib)

Note This law is similar to the one given by de Vaucoulers for the **luminosity per unit area** of elliptical galaxy as a function of r from its center.

$$\lambda(r) = \lambda(0) exp[-(\frac{r}{r_0})^{1/4}], \tag{10}$$

Astronomers like Zwicky and Smith soon realised that the total mass needed to bind the cluster by self-gravitation exceeded that present in optically luminous matter by roughly a factor of 10. From the previous section we know that the mass in the hot gas is almost equal to the luminous part, but still we are off by a factor of 5. Some proposals to counter this discrepancy are :

- **Low-mass stars**, tied to the halo of optical galaxies or loosely spread throughout the cluster.
- There exists in the cluster 10 times more stars as captured in our **current image photographs**.
- **Gas** that escapes from a galaxy would be retained by cluster owing to its large binding mass.

3.5 *Unsolved Problems Concerning Groups and Clusters*

Although most astronomers agree that the missing mass isn't really missing, and that it's just in a non-observable form – a minority of them, voiced by V.A. Ambartsumian and H.C. Arp, believe that our general of view of clusters being only gravitationally bound is wrong, and some of them maybe young systems in which individual galaxies are expelled from the cluster.

This view is counter-argued via following arguments:

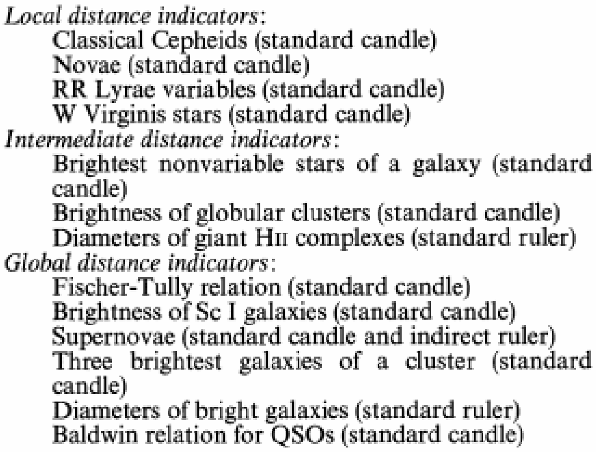
- The smooth appearance of the spatial distribution of member galaxies only suggest that these systems are in **quasi-mechanical equilibrium**.
- If they weren't gravitationally bound, the **time to disperse** would be of order 10^9 years, the time to cross cluster diameter. As the age of universe is of order $10^1 0$ years, why most of the clusters can be found in groups at present cannot be understood.
- This radical proposal would overthrow most of the established knowledge. But it is **not backed by** very convincing arguments to be accepted by other astronomers.

4 The Expansion of the Universe

Our universe is continuously expanding and this can be understood by the red-shift of the galaxies. First, we will understand the red-shift and the blue-shift: ***Red-shift and Blue-shift*** describe how light shifts toward shorter or longer wavelengths as galaxies moves. When an object moves away from us, the light is shifted to the red end of the spectrum, as its wavelengths get longer.

4.1 *The Extra-galactic Distance Scale*

Astronomers use different operations to measure the distances. They are given in the box:



Local distance indicators:
  Classical Cepheids (standard candle)
  Novae (standard candle)
  RR Lyrae variables (standard candle)
  W Virginis stars (standard candle)
Intermediate distance indicators:
  Brightest nonvariable stars of a galaxy (standard candle)
  Brightness of globular clusters (standard candle)
  Diameters of giant HII complexes (standard ruler)
Global distance indicators:
  Fischer-Tully relation (standard candle)
  Brightness of Sc I galaxies (standard candle)
  Supernovae (standard candle and indirect ruler)
  Three brightest galaxies of a cluster (standard candle)
  Diameters of bright galaxies (standard ruler)
  Baldwin relation for QSOs (standard candle)

Figure 10: *Extra-galactic Distance Indicators.*

All the methods for measuring the extra-galactic distances reduce to either the

- method of the **Standard Rulers**, or
- the method of **Standard Candles**

4.1.1 *Method of Standard Rulers*

The object of known luminosity and known distance will have a certain fraction of its light intercepted by the telescope. An object of the same luminosity at a greater but unknown distance will have a small fraction of the light intercepted by the telescope. *The ratio of the distance is inverse square root of the ratio of apparent brightness*.

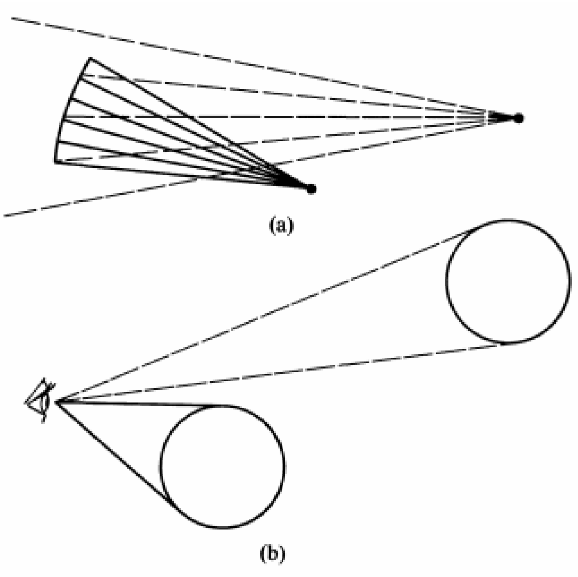


Figure 11: (a) *Method of standard candles*, and (b) *Method of standard rulers*.

4.1.2 *Method of Standard Candles*

An object of known linear size and known distance will subtend a certain angle. An object of the same linear size at a greater but unknown distance will subtend a smaller angle. *The ratio of the distances is inverse of the ratio of subtended angles*.

4.2 *Hubble's Law*

Hubble was the first to give the correct law of the expansion of the universe. He stated that the further away a galaxy is, the faster it tends to recede from us.

$$v = H_0 r, \tag{11}$$

where H_0 is the Hubble's constant, v is the velocity and r is the distance.

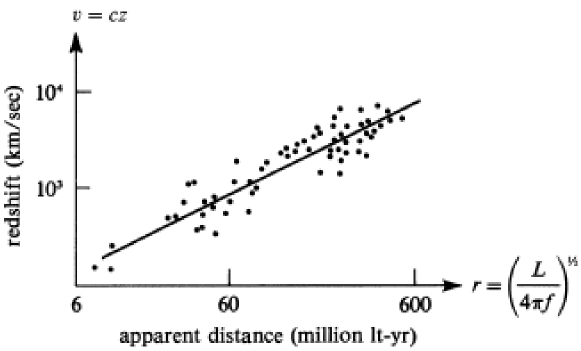


Figure 12: *Hubble diagram for Sc I galaxies. Data points well-fit around straight line, demonstrating* $v \propto r$

So, the value of H' = 15 $kmsec^{-1}/(millionlightyear)$. This value was initially obtained but on many more experimentation, an anomaly was found by Aaronson and coworkers that the **value of the Hubble's constant is different in each direction of the space**. Now an arbitrary value of the Hubble's constant is accepted as: H' = 20 $kmsec^{-1}/(millionlightyear)$.

4.3 *Naive Physical Interpretation of Hubble's Law*

Imagine that the explosion (*big bang*) occurred at the time $t = 0$, and since then time elapsed = t sec, and we are at the center of the explosion. The freely moving galaxy has travelled a distance of $v = r * t$. So, on rearranging, we have: $v = \frac{r}{t}$. Comparing it with Hubble's law, we have $H^{'} = \frac{1}{t}$. So, now $H^{'-1}$ is called as the Hubble's Time. Now, basically ignoring gravity, we will get the *exact age of the universe* = $1.5 * 10^{10}$ years. Though it later changed a bit but the formula is accurate up-to the order of magnitude. So, the universe is roughly 15 billion years old. . . **ASTONISHING!!!!!**

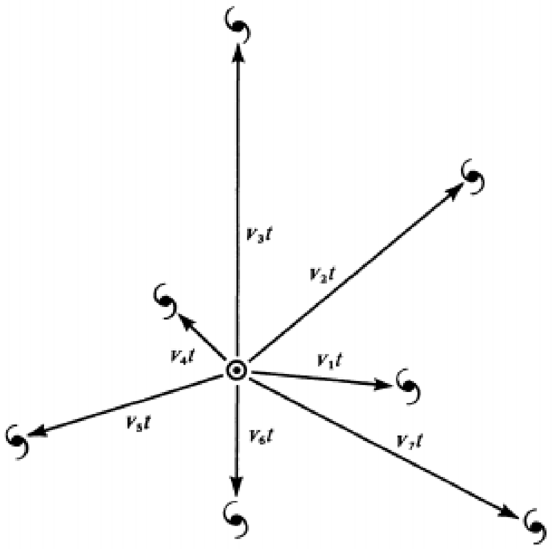


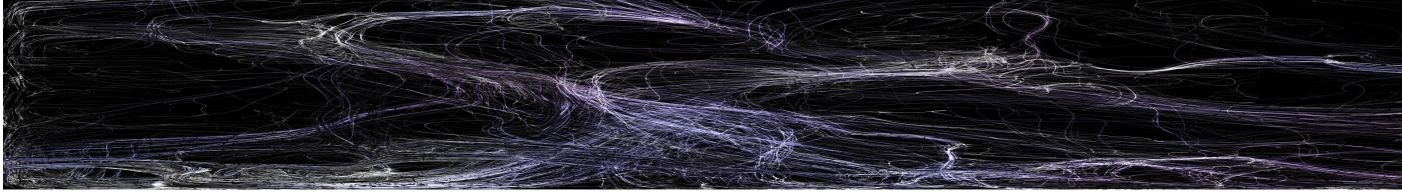
Figure 13: *Visualisation of the physical interpretation of Hubble's Law*

On the Ending Note

So we realised that even on such huge scales of galaxy clusters, it is the gravity which is dominating and playing all the game. The tendency to accumulate more and more in the centre due to self-gravitation, tidal stripping, friction due to relative motion, everywhere gravity explains the interesting phenomenon. But the universe is expanding, so does gravitation has no say at the cosmological scale?? Well, not quite.

> "
>
> *Gravitation is the very fabric of structure of space and time.*
>
> "

So, with these thoughts in mind, head on to the next topic exploring the answers – ***Gravitation and Cosmology***.



Gravitation And Cosmology

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A Basic Overview

Although we got a naive interpretation of the **Hubble's Law** earlier through an explosion plus free expansion starting at time $t = \frac{1}{7}H_0$ ago, assuming us at the center. There are two unsatisfactory aspects of the above interpretation. Firstly, the assumption of ourselves being in the center and secondly, neglecting the effects of gravitation. In the whole of this article, we will try to rectify these two shortcomings. For doing this, we will study cosmology based on Newton's perception of mechanics and gravitation i.e. an attempt to explain the astronomical observations by the principle of classical mechanics before the twentieth century. Also, we will try to attempt cosmology based on Einstein's theory of General Relativity. Hubble's Law states that: *distance to a given galaxy is proportional to the recessional velocity as measured by the Doppler's Red Shift*.

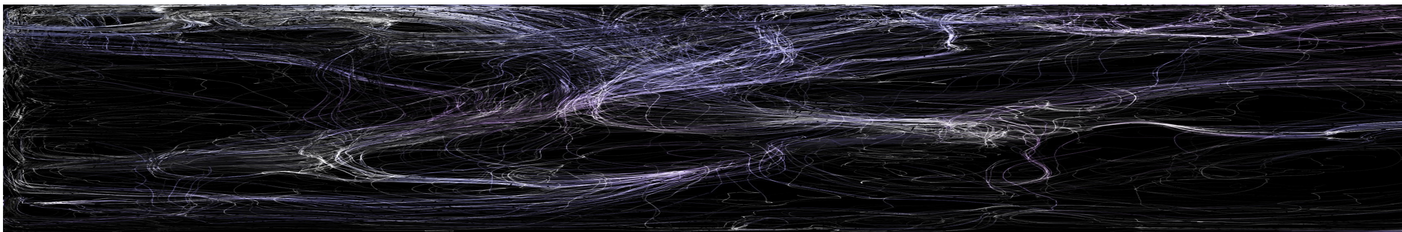
1 The Cosmological Principle.

The explanation for the expansion of the universe, which assumes us to be at the center of the expansion, is naive. There is nothing special about our position. Although few cosmologists tried to have a different perception of the same philosophically as at any instant of time, on average, we can think the universe to be homogeneous and isotropic on a large scale to any observer on a typical galaxy. The assumption that the universe is the same for all the observers at any instant is known to be the cosmological principle.

1.1 *Expansion Of Universe Using Newtonian Cosmology*

Analogy of Raisins in a Cake :

It's clear that if the universe expands, then everyone finds the same rate of expansion at a particular instant. To visualize how everyone can see everyone receding from oneself can be well understood by the analogy mentioned as: **Imagine** the raisin's situation in rising raisin cake. As the cake swells up slowly, each raisin sees every other raisin receding from itself. This seems to be incomplete because we will find that the raisins at the surface will not experience this isotropic situation of receding from itself (since no raisins on free sides). For the same, In Newtonian Cosmology Universe has to be infinite if we want to satisfy the isotropic condition of Cosmological Principle[*Refer Figure 1*]



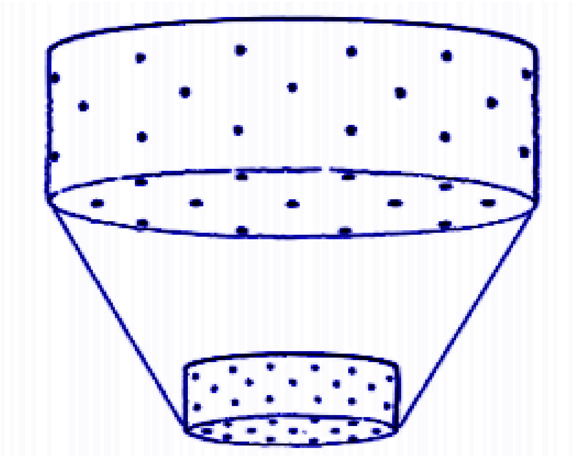


Figure 1: *A rising raisin cake illustrates a recession of raisins from each other which occurs homogeneously and isotropically.*

Using the above analogy and good understanding of the cosmological principle, we are now at the stage to draw significant conclusions.

- Firstly, that Newtonian Cosmology does require an infinite volume of space uniformly filled with galaxies.
- Secondly, that if the universe expands, then it necessarily satisfies Hubble's Law.

1.2 *Expansion Of Universe Using Hubble's Law*

Assuming Hubble's Law to be accurate, we can also show that the universe does expand uniformly and isotropically. [*Refer to Figure 2*]

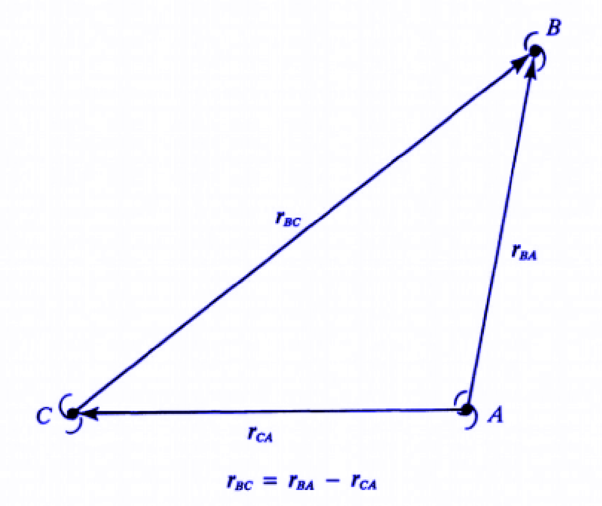


Figure 2: *Experimental Set Up diagram*

Consider the three galaxies, namely A, B, C, in the expanding universe, which satisfy Hubble's Law with respect to Galaxy A.

To show :- Hubble's Law would also apply to Galaxy C and, in extension, every other Galaxy in the universe. Applying Hubble's Law for galaxy A due to velocity formulation, we do have V_{BA}, V_{CA} for B, and C, respectively.

$$V_{BA} = H_0 r_{BA} \quad , \quad V_{CA} = H_0 r_{CA} \tag{1}$$

In Newtonian Mechanics,Velocity of Galaxy B according to galaxy C can be obtained by **vector subtraction**,

$$V_{BC} = V_{BA} - VCA = H_0(r_{BA} - r_{CA}) \tag{2}$$

But simply [r_{BA} - r_{CA}] is simply a position vector r_{BC} of galaxy B from galaxy C given by Hubble's Law:

$$V_{BC} = H_0 r_{BC} \tag{3}$$

This proves that if the universe expands according to Hubble's Law for Galaxy A, it also does for galaxy C. On a similar argument, we can derive it for almost all galaxies in the universe.

> "
> **Philosophy plays a more important role in cosmology than any other branch Of Science.**
> "

Cosmologists do preferably trust the philosophical point of view. The same happens here in Newtonian Cosmology explanation of Universe expansion, which gives rise to a significant result called Hubble's Law. We have seen very different contexts of studies of different theories and subjects. They usually have many objects or systems and varying initial conditions. **For instance**, the *Theory Of Stellar Evolution* deals with stars, which begins with different initial masses and chemical composition and theory is checked under various circumstances. However, in cosmological studies, we have only one system, the entire universe. To explain any feature about some unique system, one could postulate the feature arose because of some initials assumed. However, such an approach would be scientifically sterile. *Any explanation that relies on one assumption to produce one fact is not a real explanation; it merely rephrases the observed fact.* Therefore, cosmologists do prefer philosophical points of view, which replace a whole set of initial arbitrary conditions with a more satisfying global outlook.

2 Role Of Gravitation in Newtonian Cosmology

An individual correctly expects for the gravitation of the matter in the universe to be a significant factor for slowing down the recession of galaxies. But, the exact explanation still is a major point of view from the Newtonian Aspect. In an infinite Universe uniformly filled with matter, we do think of varieties of questions like:

- What is the direction of the gravitational field(**g**) ?
- Shouldn't on an average every matter get equal attraction from all directions ?
- Wouldn't the **gravitational field exactly be 0** everywhere in space ?
- Is gravitation a strong factor for halting the expansion of the universe sometimes in the future ?...and many more.

But, these all are so puzzling questions before us that it prevented even Newton to do something great in cosmology.

> "
> Yours and mine existence is in Question.........
> "

Surely, everyone is familiar with **Gauss Law**. This Gauss law proves Newtonian Cosmology to be an absurd conclusion. Let construct anywhere an arbitrary volume in the universe with Volume V containing the Mass M in it. Since $g = 0$ everywhere on the surface of volume V, Gauss's Law gives:-

$$4\pi GM = 0 \tag{4}$$

However, since V is entirely arbitrary, the above equation requires satisfying $M = 0$ everywhere in the universe. *In other words, the only universe satisfying the Cosmological Principle allowed by Newtonian cosmology is empty, i.e., our existence is a great question xD...................* hence we surely do agree for the disproof of the above conclusion.

3 Birkhoff's Rule for Gravitation

After all these physical and astronomical pieces of evidence, we realized that the actual difficulty is in Newton's theory of gravitation, not in the Cosmological Principle. *We can rectify the situation by replacing Gauss's Law with the Birkhoff's Rule.* Let us assume velocity v of any galaxy as seen by any observer O at a distance r away is affected only by the gravitational pull of inner galaxies inside a sphere centered at O.[*Refer to Figure 3*]

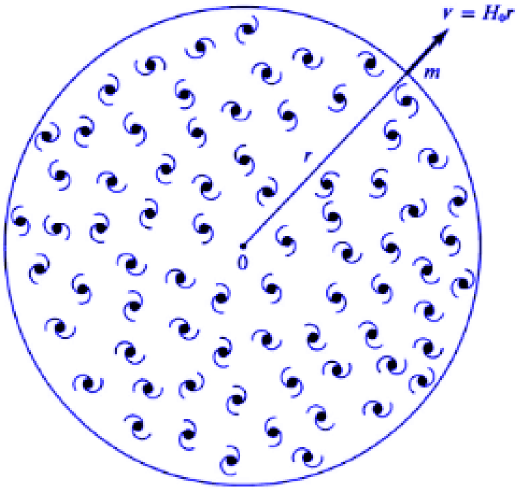


Figure 3: *-Gravitation is incorporated in Newtonian Cosmology by adopting Birkhoff's Rule.*

Assumption: Outside galaxies will not contribute any gravitational pull on mass m. Calculating deceleration of the expanding universe produced by gravitation and answering a few of the puzzling questions asked above, there is also a significant issue of whether the universe is bound or not. It can surely be investigated now by considering the energy of a representing galaxy of mass m. If average mass density in the universe currently is, the total mass M of attracting galaxies inside the radius r equals:

$$mass = density * volume = \frac{\rho_{m0} 4\pi r^3}{3} \tag{5}$$

The mass will remain the same if we follow the motion of galaxy m as it expands outward from O since all galaxies interior to it have lower recession speeds(According to Hubble's Law) and will remain interior to the expanding sphere. **The total energy of galaxy m will be conserved in the expansion process, although gravity weakens.** Given as:

$$E = \frac{mv^2}{2} - \frac{GMm}{r} = constant \tag{6}$$

Where present velocity v is given by Hubble's Law as $v = H_0 r$. And then substituting mass(M), applying boundedness and unboundedness conditions, we can find the critical case of $E = 0$ as:

$$\rho_{m0} = \frac{3{H_0}^2}{8\pi G} \tag{7}$$

Since m and r get canceled out, there is no reference to any specific galaxy and positions. From the expression mentioned above, we can quickly go through the boundedness of the universe or not.

> **Factual:** The above-mentioned critical density is almost equal to $8 * 10^{-30}\ gm/cm^3$, which is almost **equivalent to 5 hydrogen atoms per cubic meter of space**. The critical density required to bind the universe is a ridiculously small number. However, if we spread out mass in optical galaxies, we will get a smaller number, which explains the unboundedness of the universe and proof for the expansion forever, unless there are many missing masses.

4 Deceleration of the Expansion Rate

As we all got the expansion of the universe till now and found the matter density at time t must drop below its current value. Alternatively, else we can say that gravity weakens after that. So does it mean that the inequality that it holds today for

$$\rho_{m0} > \frac{3{H_0}^2}{8\pi G} \tag{8}$$

The universe is bound. Will the inequality might get reversed as the matter density drops in the upcoming nearest future?

The answer surely is **NO**. Since the boundedness condition arises from the conservation of energy and whether the universe is bound or not is solved for any particular time, it is then assumed to be fixed for all time. The energy constant E has constant numerical value; once negative, it cannot ever become positive. The fact for the declination can be focused as a point for changing the Hubble's constant. In other words, Hubble's Law should get modified as per the time variance as: $v = H(t)r$; where proportionality factor H depends upon the time. It also explains, for now, the importance of subscript 0. In the future, $H(t)$ will decrease in future, and it will even become -ve if the universe is truly bound. In this fashion:

$$\rho_{m0} < \frac{3{H_0}^2}{8\pi G} \tag{9}$$

Factual : Terminology **Hubble's Constant** may seem *inappropriate* for a quantity $H(t)$ which varies in time. However, we retain this usage because the essential aspect of $H(t)$ is that it is ideally the same value for all the observers throughout the space at any time t. Since light does not travel infinitely faster when we examine distinct galaxies, we are always looking at them as they appeared in the past, and from here, we can check for the constant's value in the past than what it is today. It also suggests it may be possible to use observations to measure the deceleration of the universe and its boundedness or not…

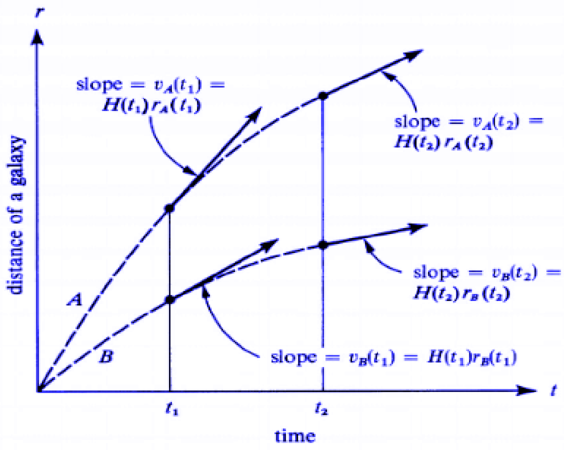


Figure 4: *[r v/s t] graph for velocity of recession of two galaxies can be depicted through the graph above.*

5 The Age of the Universe

We have constructed a naive free expansion model of a universe to estimate the age of the universe as $1/H_0$, but since gravity now acts to decelerate the expansion, it will affect the estimated age of the universe. How will it affect? We can understand this by an analogy.

> **Analogy:** Suppose you visit a friend at her house. She sees you walk in the door at 3 miles per hour, knowing that you live 3 miles away, she might deduce that you took an hour to get there. However, she would have overestimated the elapsed time if you had traveled faster in the past than when you walked through the door.

Now apply this to understand the expansion of the universe. So since gravity now acts to decelerate expansion, expansion must have been faster in the past than it is now, and the universe's actual age must be less than the free expansion value $1/H_0$. To understand this mathematically, let us take *one problem*.

> **Look through a Problem:** Consider the energy equation:
>
> $$E = \frac{mv^2}{2} - \frac{GMm}{r} = constant \tag{10}$$
>
> For the critical case $E = 0$, by taking v as dr/dt for Galaxy of mass m, we will end up with
>
> $$\Rightarrow dr/dt = (2GM/r)^{1/2}, \tag{11}$$
>
> ,where the mass M interior to r is constant throughout the expansion. By integrating this ODE and substituting $r = 0$ and $t = 0$ (big bang condition), we will have
>
> $$\Rightarrow \frac{2}{3}r^{3/2} = (2GM)^{1/2}t \tag{12}$$
>
> We also know that ,at present time $t = t_0$, $dr/dt = v = H_0 r$. By combining these two equations, We will get
>
> $$t_0 = \frac{2}{3}{H_0}^{-1} \tag{13}$$
>
> ,which is the actual age of the universe, which (we can clearly see) is less than free expansion value($1/H_0$).

6 The Fate of the Universe

The galaxies are *currently receding from us*. If the universe is unbound, galaxies will be infinitely dispersed. Eventually, all the galaxies will die, and the universe will have become empty space. This is a bleak estimate, but there is a hope. Actually there are **three possibilities**, let us understand them from the following figure [*Refer Figure 5*]

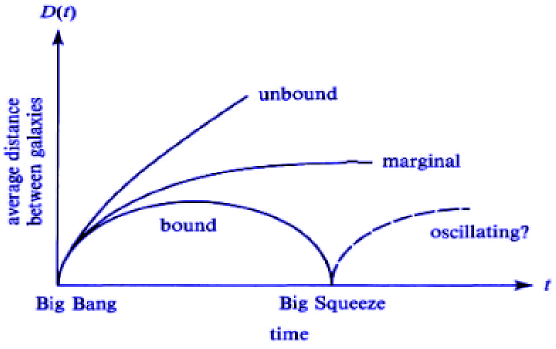


Figure 5: *History of Average separation between galaxies has been interpreted via graphical medium through the time passage under all three conditions.*

1. If the universe is unbound, the galaxies will recede forever with nonzero velocity even at the infinite separation.

2. If the universe is marginally bound, the recession velocity will be exactly zero at infinite separation.
3. If the universe is bound then, galaxies will come back together after a finite duration(big squeeze).

In the 3rd possibility, when super-high densities have been reached, will the universe then rebound in another cycle of big bang/big squeeze? Many workers agreed that our current knowledge of mechanics(*oscillating objects only oscillates about the point of equilibria*) denies that because such equilibria do not exist for conventional cosmology.

7 General Relativity And Cosmology

In cosmology, we study phenomena with the highest possible spans in space and time, so we must be careful not to think in terms of newton's concept of absolute space and absolute time. However, even special relativity is not adequate to discuss cosmology; we need **general relativity**. One example will illustrate the point. Consider the quasars, and suppose them to be at a cosmological distance. Does the cosmological principle apply to quasars? If we adopt a special relativistic interpretation and thinking phenomena when we instantaneously travel from our milky way to the nearby quasars. You will see the exact situation here that you saw at home; few quasars near you, lots far away. This is, of course, the meaning of cosmological principle; at any instant in time, all galactic observers on the average must view the same phenomena. The special relativistic explanation of the phenomena can be found in a figure.[*Refer Figure 6*]

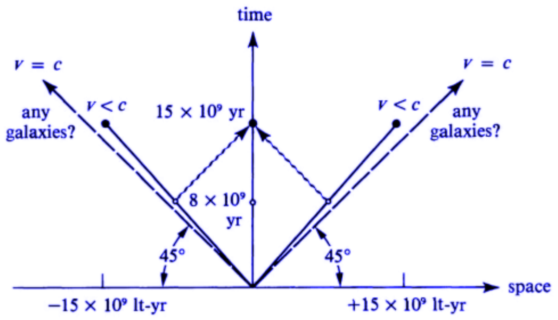


Figure 6: *Spacetime diagram of Red shift interpreted by Doppler effect.*

This figure points out a real difficulty. At this instant in time in the milky way frame of reference, are there any galaxies beyond the distance of 15 billion light-years from the milky way? If there are, and if all objects started nearly together in the big bang, how did they get so far away unless they can travel faster than the speed of light? Modern cosmologists believe that there are indeed quasars and galaxies beyond 15 billion light-years from the milky way, although they may not be observable. Moreover, they got there without having had to move faster than the speed of light relative to any local observer, indeed, in a certain sense, without having had to move at all! All this is made possible because of the space-time curvature, a concept we owe to Einstein's general relativity.

Einstein proposed the experiment about the propagation of light, where he predicted that light would travel different paths for different observers. [*Refer Figure 7*]

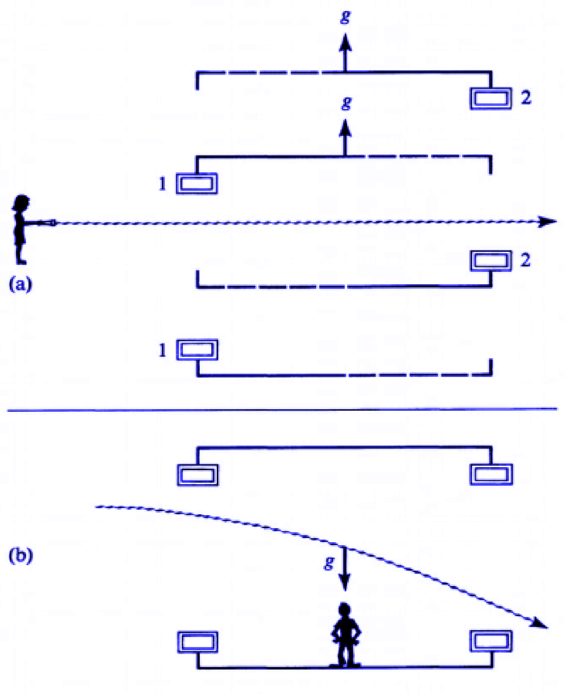


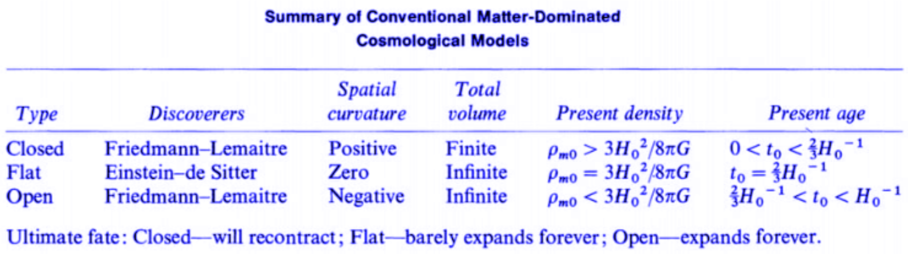
Figure 7: *Two views of experiment for propagation of light: a)Acc. to inertial observer b) Acc. to observer inside accelerating observer.*

This prediction was verified in 1919 by observing the deflection of starlight by the sun during a solar eclipse. More accurate deflection and timing experiments have been carried out on the radio emissions from quasars whose light passes near the sun so that they need not wait for a solar eclipse. If Newton had been confronted with a light bending experiment, he would probably have said that light was bent by a force of gravity. *According to Einstein, light always traveled in a straight line, and it was the space-time itself that was distorted by the presence of matter*. The **advantage of Einstein's way** of expressing it was, it became automatic that all objects should fall with the same acceleration under the influence of uniform gravitational fields. There is a disadvantage, however, which is, gravity seems different from the other forces. This difference will be an obstacle in the super-unification of the four forces of nature in the framework of relativistic quantum mechanics. Nevertheless, we will assume Einstein's geometric interpretation of gravity, which is a *correct version as far as we know*.

8 Relativistic Cosmology

Einstein's theory of gravitation was completed in 1915. Then, he adopted the perfectly natural assumption that the universe on an average was not only isotropic and homogeneous (the cosmological principle), but unchanging in time as well. But later, he found that no physically meaningful static solutions to his equations could be found except for the trivial case of an empty universe. To resolve this, *he modified his equations to incorporate a repulsive force of unknown origin, which he called the* ***cosmological constant***, balancing the gravitational effects of finite mass-energy density. However, later he renounced the introduction of the cosmological constant as **biggest blunder of his life** because it was discovered that his static models were unstable and could not remain static even if they were prepared that way initially.

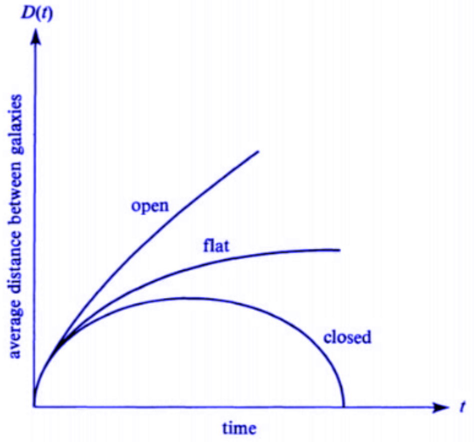
Various scientists worked over various models of the universe. Indeed it is possible to characterize the various relativistic models in terms of present density and age of the universe, as shown in the following box[*Refer Box 101*]



Summary of Conventional Matter-Dominated Cosmological Models

Type	*Discoverers*	*Spatial curvature*	*Total volume*	*Present density*	*Present age*
Closed	Friedmann–Lemaitre	Positive	Finite	$\rho_{m0} > 3{H_0}^2/8\pi G$	$0 < t_0 < \frac{2}{3}{H_0}^{-1}$
Flat	Einstein–de Sitter	Zero	Infinite	$\rho_{m0} = 3{H_0}^2/8\pi G$	$t_0 = \frac{2}{3}{H_0}^{-1}$
Open	Friedmann–Lemaitre	Negative	Infinite	$\rho_{m0} < 3{H_0}^2/8\pi G$	$\frac{2}{3}{H_0}^{-1} < t_0 < {H_0}^{-1}$

Ultimate fate: Closed—will recontract; Flat—barely expands forever; Open—expands forever.



The Differences: In Newtonian cosmology, if we have a marginally unbound universe, the addition of a hydrogen atom would make it bound. In relativistic cosmology, it is the unbound universe that has more hydrogen atoms in total than the bound universe, in fact, infinitely more in total. It is not possible to turn an unbound universe into a bound universe because they are different from the start. For geometrical reasons, cosmologists prefer to call the relativistic big bang models not bound, marginally bound and unbound. But rather *closed, flat, and open*.

As established, there are three possibilities for the relativistic big-bang models - open, closed, and flat. The closed cosmological model varies the most from the Newtonian model, and it is indeed quite non-intuitive. A Newtonian model would have to have infinite space and an infinite number of galaxies. On the other hand, the relativistic model allows finite space and a finite number of galaxies; without having boundaries or centers. This geometry is possible because the space is curved with respect to a 4th spatial dimension. This idea can be hard to understand, but if we consider a 2-dimension object curved in 3-dimensions, it becomes quite apparent. Consider a human-like ant species, who, however, can perceive only two dimensions. So consider them spread around the spherical Earth, which they imagine being flat. The world is a vast place relative to their cities. Further, let the light travel along the surface of the world only, along the shortest path between two points (the geodesic or the arc of the great circle). Furthermore, as the ant-astronomers view with their telescopes, they see other ant-cities on the surface. On counting, the number of ant cities will remain nearly constant. This observation is equivalent to the cosmological principle. They wrongly assume that since the world is flat, it has to have an edge and a center. However, we can see that the sphere has no center on the surface, and neither does it have an edge. This is true because it is curved with respect to the third dimension. However, they can realize that the surface is not flat by performing some elementary observations and calculations. [**Refer Figure 8**]

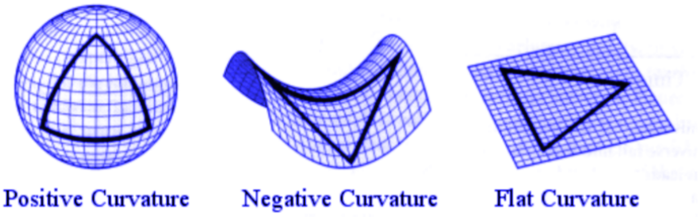


Figure 8: *All possible Curvatures.*

Now, let the radius of the radius increase, and hence, the surface area increases too. This increase is not because there is extra material for the surface but because the sphere itself grew. This idea can be analogous to our big-bang cosmological model. **The bottom line is that matter-energy distribution determines the space-time curvature**. Meanwhile, the curvature of space-time determines the motion of matter-energy.

9 Space-time curvature and explaining the apparent "Doppler effect" of the light from distant galaxies.

So, if we consider local space-time, we can have this graph.[*Refer Figure* 9]

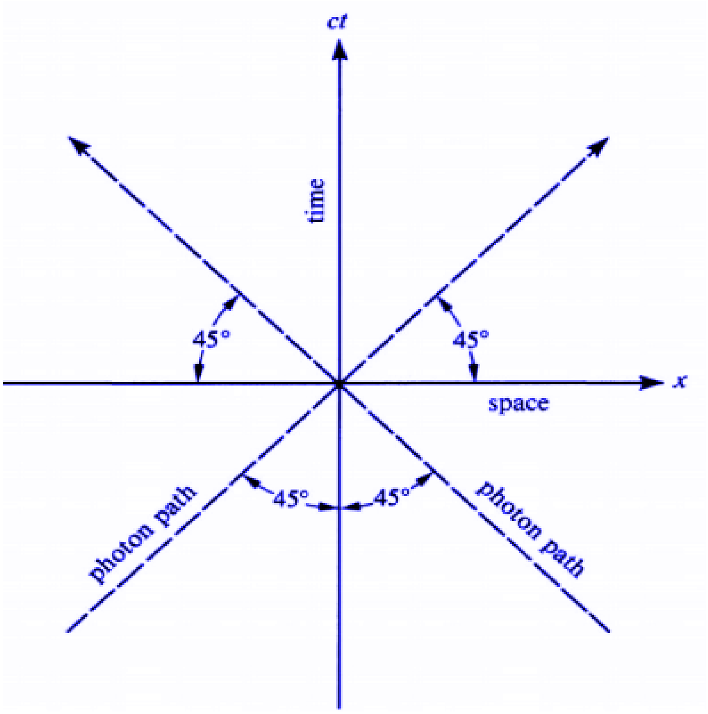


Figure 9: *The trajectories of photons in flat spacetime are at angles of 45 degrees to coordinate axes.*

We are considering only one spatial direction because of the isotropy of space. However, globally, we need to consider curved space-time. The horizontal axis represents the spatial direction while the vertical represents time. At any given point of time, space forms a loop, and the radius is the radius of curvature. In three dimensions, $2 * \pi * R(t)$ is the distance traveled in any direction to come back to the same point.[*Refer Figure 10*]

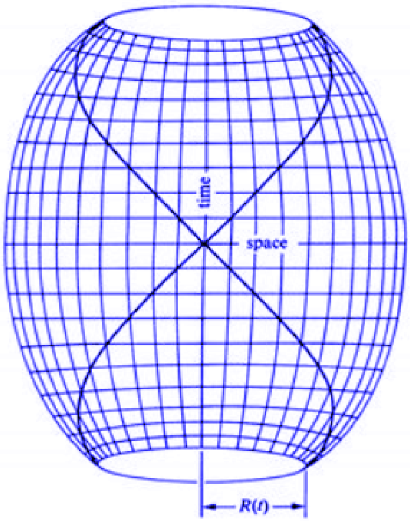


Figure 10: *Curved spacetime in a closed,matter-dominated universe during the middle half of its existence.*

Each mark in the horizontal axis is a galaxy. So, as $R(t)$ increases, the galaxies appear to move far apart, even though they are static. **Furthermore, the distant galaxies have red-shifts because of space-time curvature, not velocities of the galaxies themselves**. To understand this further, we have an example: So, if we consider there are N cells at time $t1$ and radius is $R(t1)$. Then we can define a quantity:-

$$D(t1) = 2\pi R(t1)N \tag{14}$$

At time t2, the number of cells is still N. Therefore,

$$D(t2) = 2\pi R(t2)N \tag{15}$$

Consider an electromagnetic wave emitted at time $t1$, with its wavelength equal to the length of a side of the little square at the time $t1$. The head of the wave will propagate corner to corner, and the tail vertex to vertex. This pattern will be the same, even at the time t2. So, as seen from the image,

$$\frac{lambda1}{lambda2} = \frac{D1}{D2} \tag{16}$$

. If in this epoch $D2 > D1$, that is, the universe is expanding, then $lambda2 > lambda1$. And hence a red-shift.

$$1 + z = \frac{D1}{D2}; \tag{17}$$

where z is the redshift. As z is monotonic for both distance and time, *a large z means the object is very far away*, and the light we are receiving was emitted a long time ago.[*Refer Figure 11*]

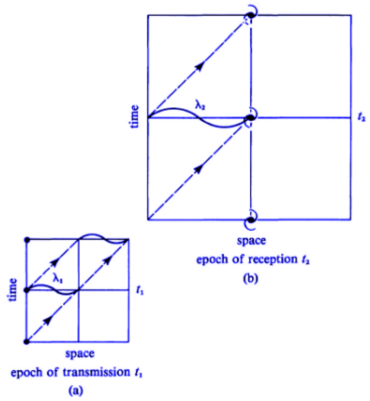


Figure 11: *Origin of Cosmological Redshift.*

> **Flat and Open universe:** The main difference between the open/flat universe and the closed universe would be that these have infinite space and an infinite number of galaxies. However, within a finite time after the Big Bang, we can only see a finite number of galaxies due to the limit on the speed of the light from the farther galaxies. The geometries of an open universe are more intuitive than closed because the density of matter-energy less warps space-time. Also, as the time after the big bang tends to infinity, the density of energy and matter drops to zero. The universe resembles the flat space-time of special relativity.

10 Cosmological Tests through Hubble diagram

Let the luminosity of a galaxy be L in its rest frame, and it emits photons isotropically into space. Consider an observer at a different location and after a specific time. So we need to calculate the apparent intensity f, considering the cosmological effects. Since we are assuming a non-flat universe, the distance between the galaxy and observer will be a function of time, say $r(z)$, where z is the redshift. Furthermore, if we consider a sphere of radius $r(z)$, for a large z,

$$f < \frac{L}{4\pi(r(z)^2)} \tag{18}$$

because of the expansion. The energy of all photons is reduced by a factor $(1+z)^{-1}$. This is because the wavelength increases, and hence, the energy decreases. The equations are:

$$1 + z = \frac{D1}{D2} = \frac{l1}{l2} \tag{19}$$

$$E = \frac{hc}{l} \tag{20}$$

where h is Planck's constant, c is the speed of light. Moreover, the time required to receive one wave-train of photons has also increased by the factor of $(1+z)$. Therefore,

$$f = \frac{L}{4\pi(r(z)^2)(1+z)^2} \tag{21}$$

Here, $r(z)$, depends on the model we are assuming. So the plot between $r(z)$ and z can give us some conclusive proof. However, this is not a full-proof idea. The main errors can occur because of other effects seen in the galaxies, like evolution and cannibalism seen in galaxies. Evolution means that the luminosity of the galaxy was intrinsically higher in the past. So this can falsely imply that the open model fits the data. On the other hand, cannibalism implies that some galaxies are getting brighter and they might falsely imply the closed model fits the data. So, while large values of z can give conclusive evidence for a model, there are also significant effects of galaxies' interactions, which can drastically affect the plot. [*Refer Figure 12*]

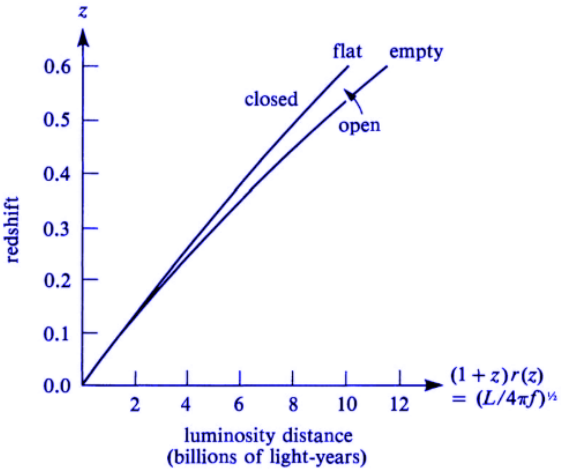


Figure 12: *Theoretical Hubble diagram in which galaxies of a given luminosity L but of different apparent brightness f and redshift z in open universe would lie between the curve labelled "flat" and "empty" while galaxies in a closed universe would lie above and to left of "flat".*

On the Ending Note

Concluding the entire chapter through a smooth note ,we have learned that at the largest scales of time, gravitation dominates the scene of cosmology. **The Universe expands, not because galaxies are moving away from us(red shift) but because of expansion of space and time themselves.**In particular, according to Einstein's geometric interpretation of gravitation , the very fabric of spacetime in the universe is manufactured by gravitation .This explains why our conception of the physical world is often referred as **Einstein's Universe**. *The main astronomical issue to arise from this insight is whether the universe is open or closed*. If the present average mass density is low ,the universe is open, and there exists a finite amount of space and a finite number of galaxies. If the universe is open, the galaxies will recede forever.If the universe is closed, the galaxies will be reforged in fiery crucible. The above arguments evidence the universe to be open, but our hearts hope for its closure.

So you have enough gobbled yourself with theory and a pinch of astronomical physics, so why not divert and gear up yourself with one of the fundamental and cool mathematics tool with heavy application in astronomy, electromagnetism, mechanics and the list goes on and on and on: ***Tensors.*** The next section would carry you from its different definitions to different transformation rules, and end on its real applications.

Tensors

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Introduction

The concept of Tensors is the basic one required if one wants to fully understand the Mathematics behind the content we've covered in the project.
Let us start with some basic definitions:
In an m-dimensional space, a tensor of rank n is a mathematical object that has n indices, mn components and obeys certain transformation rules.
Where, Rank is the number of basis vectors required to fully specify a component of the Tensor.
To give you a basic idea of a tensor, here's an example:

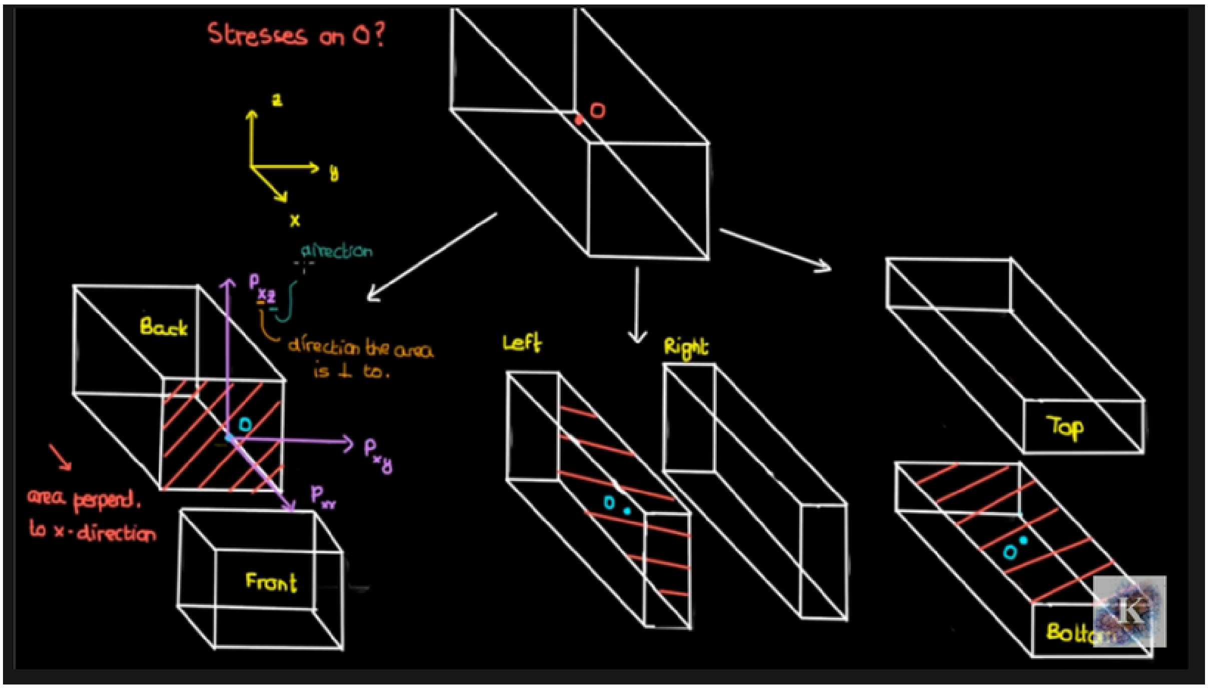

Figure 1: *In the picture above, we try to calculate the stress on point O in the beam. So, to understand this, we slice the beam in three ways possible to find out the Stress.In the first case, note that* P_{xx} *creates a normal stress on point O, since it is perpendicular to the area. While in the second case* P_{yx} *(Not* P_{xy}*) creates a shear stress in the same direction. Thus, these stress can't be added together as the same quantity. Thus, each slice has 3 different components of Stress, giving us a total of 9 components. This Stress is thus represented in the form of a 3x3 Matrix.*

Going by the definition, a tensor follows certain transformation rules. A tensor never changes by the change of the coordinate system (it is **invariant**). However the components of the tensor (the magnitude and the basis vectors of each component) undergo a change following a special set of mathematical rules. These rules help us to understand and handle tensors in different coordinate systems. However it is important to understand the basics of Einstein notation to understand these rules in a better way.

1 Einstein Notation

In Mathematics, especially in applications of linear algebra to physics, the Einstein notation or Einstein summation convention is a notational convention that implies summation over a set of indexed terms in a formula, thus achieving notational brevity. What this means, is that it is a new representation of Summation so that the equations we work with using tensors may look neat.
Index: a_i and a^i

1.1 The rules to the notation

1. Any twice repeated index in a single term is summed over. e.g. $a_{ij}b^j = a_{i1}b^1 + a_{i2}b^2 + a_{i3}b^3$, (usually the indexing is 1 to 3 because we deal with a 3-D space).
   'j' is called the **dummy index** (Repeated Twice), and 'i' is called the **free index**. So, we can replace j with any other character because at the end it has to be summed over.
2. There are two indices, Superscript and Subscript, which account for the Contravariant and Covariant components of a tensor respectively.
3. No Index may occur 3 or more times in a given term. When we count the indices, we count both the subscript and superscript indices. e.g. $a_{ij}b^j$, j repeated twice, rule not violated.
4. In an equation involving Einstein Notation, the free indices on both sides must match.
   e.g. $x_i = a_{ij}b^j$, both sides have i as free index, so the rule is not violated.

1.2 Some Non-Identities

1. $a_{ij}(x_i + y_j)$ $a_{ij}x_i + a_{ij}y_j$
2. $a_{ij}x_iy_j$ $a_{ij}x_jy_i$
3. $(a_{ij} + a_{ji})x_iy_j$ $2a_{ij}x_iy_j$

1.3 Some Identities

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2. $a_{ij}x_iy_j = a_{ij}x_jy_i$
3. $a_{ij}x_ix_j = a_{ji}x_ix_j$
4. $(a_{ij} + a_{ji})x_iy_j = 2a_{ij}x_iy_j$
5. $(a_{ij} - a_{ji})x_ix_j = 0$

1.4 Some Identities

In mathematics, the **Kronecker delta** is a function of two variables, usually just non-negative integers. The function is 1 if the variables are equal, and 0 otherwise.

$$\delta^i_j = 1 \quad \text{for i=j}$$

$$\delta^i_j = 0 \quad \text{otherwise}$$

2 Coordinate Transformation For Tensors

Let $\vec{x}$ be a vector in C coordinate system and $\vec{\overline{x}}$ be its corresponding vector in C' coordinate system, then in general,

$$\vec{\overline{x}} = T(\vec{x})$$

Here T is a set of functions that defines the transformation. Therefore for each possible i, we can write,

$$\overline{x}^i = T^i(x^1, x^2, \ldots.., x^n) \quad \forall \quad 1 \leq i \leq n$$

If the functions $T^i(x^1, x^2, \ldots.., x^n)$ are all real valued, have continuous second partial derivatives in the domain $\left(\frac{\partial^2 \overline{x}^i}{\partial (x^j)^2} \quad \forall \quad 1 \leq i, j \leq n\right)$ and are all invertible, then, the transformation T is called a coordinate transformation.
Also, if the (x^i) represents a rectangular coordinate system (say C) and the functions T transform it into $(\overline{x}^i)$ which represents C', then C' is called
i) Affine if T is linear,
ii) Curvilinear if T is non-linear

2.1 An example of transformation

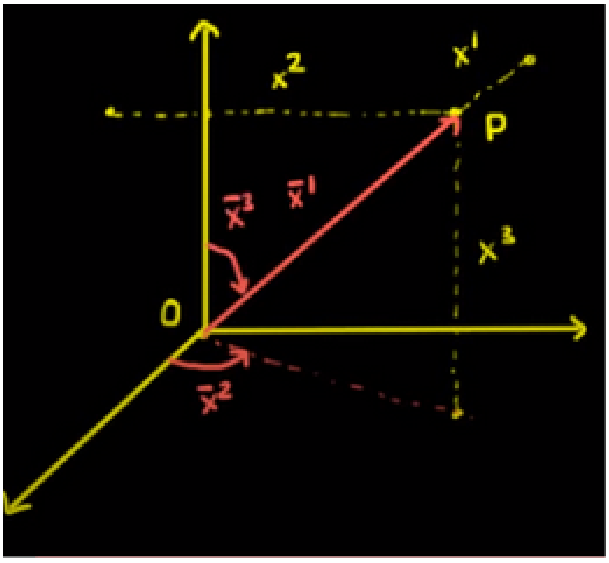

Figure 2: *From linear to spherical (curvilinear) coordinate system.*

In the rectangular coordinate system, let there be a point $P : (x^1, x^2, x^3)$ such that x^1, x^2, $x^3 \in \mathbb{R}$. Therefore, in spherical system, we have,

$$\overline{x}^i = T^i(x^1, x^2, x^3)$$

$$\overline{x}^1 = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}$$

$$\overline{x}^2 = \tan^{-1}\left(\frac{x^2}{x^1}\right)$$

$$\overline{x}^3 = \cos^{-1}\left(\frac{x^3}{\sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}}\right)$$

i) Observe that $T^i(x^1, x^2, x^3)$ is a real valued function.
ii) $\frac{\partial^2 \overline{x}^1}{\partial (x^1)^2} = \frac{(x^2)^2 + (x^3)^2}{\sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}}$, which is continuous everywhere except the origin.

Similarly, upon working out, we can find that all the second partial derivatives $\left(\frac{\partial^2 \overline{x}^i}{\partial (x^j)^2} \quad \forall \quad 1 \leq i, j \leq 3\right)$ are continuous everywhere (except origin). Also,

$$x^1 = \overline{x}^1 \cos(\overline{x}^2) \sin(\overline{x}^2)$$

$$x^2 = \overline{x}^1 \sin(\overline{x}^2) \sin(\overline{x}^3)$$

$$x^3 = \overline{x}^1 \cos(\overline{x}^3)$$

This means that $(T^i)^-$ exists for all T^i $(1 \leq i \leq 3)$. Therefore the above transformation is a coordinate transformation. Also, we see that the T^i (all 3 in this case) are non-linear. Therefore the spherical coordinate system is a curvilinear system.

2.2 Jacobian

Suppose $y^i = T(x^1, x^2, x^3, ..., x^n)$ is a coordinate transformation defined by T. The Jacobian(say ν) for this transformation is defined as the determinant of the Jacobian matrix.

$$J = \begin{bmatrix} \partial y^1/\partial x^1 & \partial y^1/\partial x^2 & \cdots & \partial y^1/\partial x^n \\ \partial y^2/\partial x^1. & . & \cdots & . \\ \vdots & \vdots & & \vdots \\ \partial y^n/\partial x^1 & . & \cdots & \partial y^n/\partial x^n \end{bmatrix}$$

Figure 3: *The Jacobian matrix.*

Therefore, $\nu = |J|$ Jacobian is an important mathematical tool. One of the many uses of Jacobian is to determine whether a given transformation is bijective (i.e one to one).
Also, J is jacobian matrix of T J^{-1} is the jacobian matrix of T^{-1}.

2.3 Tensor Rank

Suppose V is a tensor with components $V^{i_1 i_2 \ldots i_p}_{j_1 j_2 \ldots j_q}$. Then the rank of V is $m = p + q$.
E.g. All these are rank 2 tensors, a_{ij}, A^{ij}, A^i_j.

3 Contravariant and Covariant Vector Components

Contravariant Component denoted by a^i.
Covariant Component denoted by b_i.

Assumptions: Suppose V is a vector field defined on a subset of $\mathbb{R}$, and suppose that x^i and $\overline{x}^i$ are two coordinate systems related by the coordinate transformation (T):

$$\overline{x}^i = \overline{x}^i(x^1, x^2, x^3, ..., x^n)$$

3.1 Contravariant Vector

The vector field V is said to be a contravariant tensor of rank 1 if its components V^i in the x^i coordinate system and $\overline{V}^i$ in the $\overline{x}^i$ coordinate system are related by the following law of transformation:

$$\overline{V}^i = V^r \frac{\partial \overline{x}^i}{\partial x^r} \quad (1 \leq i \leq n)$$

where ris the dummy index.

We can extend the idea for rank 2 tensors as follows: Suppose that V is now a matrix field composed of n x n scalar fields (functions) defined over a region in $\mathbb{R}^n$.
Then V is a contravariant tensor of rank 2 if its components V^{ij} in the x^i coordinate system and $\overline{V}^{ij}$ in the $\overline{x}^i$ coordinate system are related by the following law of transformation:

$$\overline{V}^{ij} = V^{rs}\frac{\partial \overline{x}^i}{\partial x^r}\frac{\partial \overline{x}^j}{\partial x^s} \quad (1 \leq i, j \leq n)$$

where r,sare the dummy indices.

3.2 Covariant Vector

(We take similar assumptions for Covariant Vector as well) The vector field U is said to be a covariant tensor of rank 1 if its components U_i in the x^i coordinate system and $\overline{U}_i$ in the $\overline{x}^i$ coordinate system are related by the following law of transformation:

$$\overline{U}_i = U_r\frac{\partial x^r}{\partial \overline{x}^i} \quad (1 \leq i \leq n)$$

Note: The order of the partial derivative is switched.
We can extend the idea for rank two tensors as follows: Suppose that U is now a matrix field composed of n x n scalar fields (functions) defined over a region in $\mathbb{R}^n$. Then U is a covariant tensor of rank 2 if its components U_{ij} in the x^i coordinate system and $\overline{U}_{ij}$ in the $\overline{x}^i$ coordinate system are related by the following law of transformation:

$$\overline{U}_{ij} = U_{rs}\frac{\partial x^r}{\partial \overline{x}^i}\frac{\partial x^s}{\partial \overline{x}^j} \quad (1 \leq i, j \leq n)$$

where r,sare the dummy indices.

3.3 Mixed Tensor

It has both contravariant and covariant indices.
A matrix field Z is a mixed tensor of rank 2 if its components Z^i_j in the x^i coordinate system and $\overline{Z}^i_j$ in the $\overline{x}^i$ coordinate system obey:

$$\overline{Z}^i_j = Z^r_s\frac{\partial \overline{x}^i}{\partial x^r}\frac{\partial x^s}{\partial \overline{x}^j} \quad (1 \leq i, j \leq n)$$

where r,sare the dummy indices.

3.4 Problems

1. If A^i and B_j are tensors, prove that $C^i_j = A^iB_j$is a tensor.
2. If A^{ij} and B_{jk} are tensors, prove that $A^{ij}B_{jk}$ is some tensor C^i_k.

4 Line Element and Metric Tensor

In space V_n, between two points x^i and $x^i + dx^i$ the distance ds is

$$ds^2 = g_{ij}dx^idx^j$$

This quadratic differential form is called the Riemannian Metric or line element for n-dimensional space, space is called Riemannian Space, and g_{ij} is called Metric Tensor or Fundamental Tensor.

Theorem: The Metric Tensor g_{ij} is a covariant symmetry tensor of rank two.
Proof:
The distance between two neighbouring points is independent of the coordinate system. That is, ds^2

invariant. From the quotient law, it follows that $g_{ij} + g_{ji}$ is a covariant tensor of second order. We can write

$$g_{ij} = \frac{1}{2}(g_{ij} + g_{ji}) + \frac{1}{2}(g_{ij} - g_{ji})$$

The contribution of $\frac{1}{2}(g_{ij} - g_{ji})dx^i dx^j$ to ds^2 is zero, hence there is no loss of generality in assuming that g_{ij} is symmetric. Thus g_{ij} is a covariant symmetry tensor of rank two.

Theorem: $g_{ij}dx^i dx^j$ is an invariant.

Proof:

Let x^i be coordinates of a point in X-coordinate system and x^j be coordinates of the same point in Y-coordinate system. Since g_{ij} is a Covariant tensor of rank two. Then,

$$\overline{g}_{ij} = g_{kl}\frac{\partial x^k}{\partial \overline{x}^i}\frac{\partial x^l}{\partial \overline{x}^j}$$

$$\overline{g}_{ij} - g_{kl}\frac{\partial x^k}{\partial \overline{x}^i}\frac{\partial x^l}{\partial \overline{x}^j} = 0$$

$$\left(\overline{g}_{ij} - g_{kl}\frac{\partial x^k}{\partial \overline{x}^i}\frac{\partial x^l}{\partial \overline{x}^j}\right) dx^i dx^j = 0$$

$$\overline{g}_{ij}dx^i dx^j = g_{kl}\frac{\partial x^k}{\partial \overline{x}^i}\frac{\partial x^l}{\partial \overline{x}^j}dx^i dx^j$$

$$\overline{g}_{ij}dx^i dx^j = g_{kl}dx^i dx^j$$

So, $g_{ij}dx^i dx^j$ is an invariant.

4.1 Conjugate Metric Tensor (Contravariant tensor)

The conjugate metric tensor to g_{ij}, which is written as g^{ij} , is defined by

$$g^{ij} = \frac{B_{ij}}{g}$$

Where B_{ij} is the cofactor of g^i in the determinant $g = |g_{ij}| \neq 0$

By theorem,

$$A^{ij}A_{ik} = \delta^i_k$$

So,

$$g^{ij}g_{jk} = \delta^i_k$$

Note:

(1) Tensors g_{ij} and g^{ij} are Metric Tensor or Fundamental Tensors.

(2) g_{ij} is called the first fundamental Tensor and g^{ij} is called second fundamental Tensors.

4.2 Problems

1. Find the Metric and component of the first and second fundamental tensor in spherical coordinates.
2. Given the metric tensor and inverse metric tensor g_{ij} and g^{ik}, we know that $g_{ij}g^{ik} = \delta^k_j$. Using this property, show that

$$g_{ij}A^i = A_j$$

And

$$g^{ij}B_i = B^j$$

4.3 Length of a Curve

Consider the curve $x^i = x^i(t)$ with the parameter t. The length of the curve between the points $t = t_1$ and $t = t_2$ is given by

$$S = \int_{t_1}^{t_2} \sqrt{g_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt}} dx$$

4.4 Null Curve

If $g_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt} = 0$ along a curve. Then $s = 0$. Then the points $P1$ and P_2 are at zero distance, despite the fact that they are not coincident. Such a curve is called a minimal curve or null curve.

4.5 Associated Tensor

A tensor obtained by the process of the inner product of any tensor $A^{i_1 i_2 \dots i_r}_{j_1 j_2 \dots j_s}$ with either of the fundamental tensors g_{ij} or g^{ij} is called associated tensor of the given tensor. e.g. Consider a tensor A_{ijk} and form the following inner product

$$g^{\alpha i} A_{ijk} = A^{\alpha}_{j\ k} \quad ; \quad g^{\alpha j} A_{ijk} = A^{i}_{\alpha k} \quad ; \quad g^{\alpha k} a_{ijk} = A^{i}_{j \alpha}$$

All these tensors are called associated tensors of A_{ijk}

Associated Vector

Consider a covariant vector A_i . Then $g^{ik} A_i = A^k$ is called associated vector of A_i. Consider a contravariant vector A^j. Then $g_{ik} A^j = A_k$ is called the associated vector of A^j.

4.6 Magnitude of a Vector

The magnitude or length A of contravariant vector A^i. Then A is defined by

$$A = \sqrt{g_{ij} A^i A^j}$$

$$A^2 = g_{ij} A^i A^j$$

The magnitude or length A of covariant vector A_i. Then A is defined by

$$A = \sqrt{g^{ij} A_i A_j}$$

$$A^2 = g^{ij} A_i A_j$$

The magnitude of the two associated vectors are equal.

Angle Between Two Vectors

The angle between two unit vectors A^i and Bi is defined by

$$\cos(\theta) = g_{ij} A^i B^j = A_j B^j = g^{jk} A_j B_k = A^k B_k$$

5 Tensor Operations

These are some basic operations on the General Tensor.
Suppose S and T are tensors;

$$S \equiv S^{i_1 i_2 \dots i_p}_{j_1 j_2 \dots j_q}$$

$$T \equiv T^{k_1 k_2 \dots k_r}_{l_1 l_2 \dots l_s}$$

S has $p$contravariant and q covariant, while T has r contravariant and s covariant components.

1. **Summation**

   If, $p = r$ and $q = s$, then,

   $$S + T \equiv (S^{i_1 i_2 .... i_p}_{j_1 j_2 .... j_q} + T^{k_1 k_2 .... k_r}_{l_1 l_2 .... l_s})$$

   Note that $S + T$ and $S - T$ have the same rank as S and T.

2. **Scalar Multiplication**

   $(S\mu)$ where μ is a constant is, $(\mu * S^{i_1 i_2 .... i_p}_{j_1 j_2 .... j_q})$. The new Tensor has the same rank.

3. **Linear Combination**

   If T_1, T_2,... T_u are tensors of the same ranks, then $\mu_1 T_1 + \mu_2 T_2 + \ldots + \mu_u T_u$ is also a tensor of the same rank.

4. **Conraction**

   As stated above, S is a (p, q) tensor. The contraction of S w.r.t. a contravariant index i_f and a covariant index j_g is given by setting $i_f = j_g = u$, and evaluating,

   $$S^{i_1 i_2 .., u, .. i_p}_{j_1 j_2 .., u, .. j_q} = \sum_{u=1}^{3} S^{i_1 i_2 .. j_{f-1}, u, j_{f+1} .. i_p}_{j_1 j_2 .. j_{g-1}, u, j_{g+1} .. j_q}$$

   since u becomes a dummy index. S’ is now a (p-1, q-1) tensor.

5. **Inner Product**

   Choose a contravariant index on S (i_f), and covariant index on T (l_g). Let, $i_f = l_g = u$.

   $$(S.T) = (S^{i_1 i_2 .... i_p}_{j_1 j_2 .., u, .. j_q} * T^{k_1 k_2 .., u, .. k_r}_{l_1 l_2 .... l_s})$$

   where u is dummy, therefore summed over. So, $(S.T)$ is a tensor of Rank $p + q + r + s - 2$.

6. **Outer Product**

   $$(ST) = S^{i_1 i_2 .... i_p}_{j_1 j_2 .... j_q} * T^{k_1 k_2 .... k_r}_{l_1 l_2 .... l_s}$$

   with rank $p + q + r + s$, a (p+r, q+s) Tensor.

BIBLIOGRAPHY

`Primary Book:  The Physical universe by Frank Shu`

- **Big Bang**
    1. **Secondary book: Jonathan Allday - Quarks, leptons, and the big bang**
    2. https://en.wikipedia.org/wiki/Cosmological_principle
    3. https://www.britannica.com/science/cosmology-astronomy/Relativistic-cosmologies
    4. (image) https://en.wikipedia.org/wiki/Big_Bang
    5. (image) https://thetechreader.com/top-ten/top-ten-scientific-flaws-in-the-big-bang-theory/
    6. https://en.wikipedia.org/wiki/Grand_unification_epoch
    7. http://www.astronomycafe.net/FAQs/q1050x.html
    8. (GIF) https://www.youtube.com/watch?v=yArprk0q9eE
    9. (image)https://sciencevspseudoscience.files.wordpress.com/2014/03/reflections.png
    10. (image) https://miro.medium.com/max/700/0*lSM6PBKd45CcwX7B.
    11. https://www.symmetrymagazine.org/article/charge-parity-violation
    12. https://en.wikipedia.org/wiki/Baryon_asymmetry
- **Clusters of Galaxies and the Expansion of the Universe**

References

[1] Book : The Physical Universe – *Frank H. Shu*

[2] Link Of Reference for Hubble's Law and Expanding Universe
`https://www.pnas.org/content/112/11/3173`

[3] de Vancouleurs Law
`https://en.wikipedia.org/wiki/De_Vaucouleurs%27_law`

[4] This Is How Big The Local Group of Galaxies Is
`https://youtu.be/GW2a9xwpd50`

[5] Hubbles' Law and the Expanding Universe
`https://www.pnas.org/content/112/11/3173`

[6] Hubbles'Law — Khan Academy
`https://youtu.be/1V9wVmO0Tfg`

1 tensors

[7] Book : Tensor Calculus – *Barry Spain*

[8] Introducing Tensors
`https://www.youtube.com/watch?v=uaQeXi4E7gA`

[9] Tensor - Wikipedia
`https://en.wikipedia.org/wiki/Tensor`

[10] Kronecker Delta - Wikipedia
`https://en.wikipedia.org/wiki/Kronecker_delta`

[11] Contravariance and Covarinace
`https://www.youtube.com/watch?v=vvE5w3iOtGs`

[12] Introduction to Tensors: Transformation Rules
`https://youtu.be/j6DazQDbEhQ`

[13] Coordinate Transformations and Curvilinear Coordinates — Tensor Calculus
`https://youtu.be/XtpVVcKXfnA`

2 Gravitation And Cosmology

[14] Book : The Physical Universe –Frank H. Shu

[15] Link Of Reference for Hubble's Law and Expanding Universe
`https://www.pnas.org/content/112/11/3173`

[16] Newtonian Cosmology Interpretation
`http://www.iucaa.in/ somak/courses/extgal1/Newtonian-cosmology.pdf`

[17] Deceleration of the expanding universe
`https://arxiv.org/pdf/1502.00811`

[18] Age of the Universe
`https://en.wikipedia.org/wiki/Age_of_the_universe`

[19] Relativistic Cosmology
`http://strangebeautiful.com/other-texts/ellis-maartens-maccallum-relativistic-cosmo.pdf`

[20] Hubble Diagram Test of Expanding and Static Cosmological Principle
`https://www.hindawi.com/journals/aa/2013/917104/`

[21] Introduction to Cosmology
`http://gravitation.web.ua.pt/sites/default/files/migrated2016/Lecture_2.pdf`

[22] Space Time Curvature
`https://www.science.org.au/curious/space-time/gravity`